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An Investigation of the Applicability of the Streamer Theory of Spark Discharge to the Problem of Extinguishing the Short Alternating Current Arc.

by

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Academic Dean

PREFACE

The purpose of this paper is to attempt a theoretical investigation of the conditions existing in the gap between electrodes of a short alternating current arc immediately after the current passes through its zero value and to apply a recently developed theory of spark discharge, modified to fit these conditions, in an effort to explain the phenomenon of re-ignition. In so doing, the authors hope to indicate how best to suppress the arc by possible modifications of the normal gap conditions.

The authors wish to express their grateful appreciation for the guidance and encouragement given by Doctor C. V. O. Terwilliger at times when it appeared that the lack of adequate knowledge of some of the basic physical processes involved would prevent any solution of this problem.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Page

TABLE OF SYMBOLS

c	average velocity
e	electronic charge (4.803×10^{-10} e.s.u.)
f	average fraction of energy lost by a particle at impact
i	instantaneous electric current
k	mobility
m	electronic mass ($.91 \times 10^{-27}$ grams)
n	number (particles, collisions, etc.)
n'	attachment constant (average number of electron impacts which result in one attachment to form a negative ion)
p	pressure (millimeters of mercury)
r	radial distance of diffusion
t	time
v	average drift velocity in field direction
x	distance from cathode in field direction
B	rate of increase of impressed voltage
C	root mean square velocity
D	coefficient of diffusion
E	average kinetic energy
L	thickness of cathode space charge
M	mass of gaseous particles
N	density (particles per unit volume)
T	temperature
V	potential at point distant x from cathode at time t
V_o	potential impressed on electrodes

$\underline{\underline{X}}$	field intensity
$\underline{\underline{X}}'$	field intensity due to space charge built up by an electron avalanche
α	coefficient of ionization by electron impact
α_r	coefficient of recombination
δ	gap length
ϵ	base of Napierian (natural) logarithm
λ	mean free path
σ	radius of a particle
σ_{12}	sum of radii of colliding particles

INTRODUCTION

1. The role of the arc.

A circuit breaker in any electrical circuit must perform a dual role. When closed, it must function as a good conductor and pass hundreds or even thousands of amperes with only a few volts drop across it. When open, it must withstand hundreds or thousands of volts without permitting any but minute currents of the order of a few milli-amperes. To do this, it must change from a good conductor to a good insulator with a resistance of hundreds of thousands of ohms.

All practical electrical circuits contain inductance, capacitance, and resistance. Therefore, when a current flows, electro magnetic energy is stored in the circuit. If destructive reactions are to be avoided when the circuit is opened and the flow of current interrupted, this energy must be dissipated. The arc performs this function of energy dissipation. It is interesting to note that if the arc did not automatically form when the contacts of a circuit breaker were separated, it would be necessary to invent its equivalent to obtain satisfactory circuit interruption.

The circuit breaker must provide the arc and then by some means convert this highly conducting path into a medium of high dielectric strength. Modern power distribution systems require that this change be made in a matter of a few micro seconds.

2. The air circuit breaker.

In recent years, more and more attention has been directed toward the air circuit breaker as the device capable of performing the required functions without many of the problems encountered in using other media. Although many investigators have examined the various phenomena involved, no complete theory has as yet been developed that will satisfy all the conditions. It was this fact that led the authors of this paper to investigate from the theoretical point of view the interruption of alternating currents in short arcs in open air.

3. The scope of the investigation.

This paper is limited to a study of the arcs produced with small electrode spacing by alternating currents in open air.

The problem seemed to divide itself naturally into four main divisions; the development of an adequate spark theory, the determination of the requirements imposed by the external circuit, the determination of the conditions existing in the gap at the time of and immediately after the current zero, and the application of the spark theory under the conditions determined.

The streamer theory of spark discharge developed by Loeb and Meek (8) seemed most likely to afford an adequate explanation of the observed phenomena. An examination of this theory as slightly modified to fit the assumed conditions showed that the critical breakdown voltage that would just cause the arc to re-ignite was a function of the field strength at points across the gap between the electrodes. However, the lack of data on the coefficient of ionization by electron impact which is an integral part of this theory forced the authors to make arbitrary assumptions as to the temperature in the region where the arc had existed. Since this assumed temperature is undoubtedly low, the results obtained can be considered correct only in a qualitative sense.

Since the streamer theory requires a knowledge of the voltage gradient at all points in the gap, an expression was developed for the field strength in terms of the voltage recovery rate of the external circuit, the coefficient of recombination of ions in air, the mobility of positive ions in an electrical field, and the time interval after the current zero. Here again, the lack of adequate data on the values of the mobility and coefficient of recombination and particularly the manner of variation of these with temperature detracted from the quantitative value of the results.

The rate of recovery of the voltage impressed upon the electrodes was determined by the application of conventional transient circuit analysis.

With the voltage gradient at all points determined, it was possible to apply the theory of spark discharge and thereby determine the maximum voltage that a gap would sustain without the arc re-igniting under a particular rate of recovery of the voltage of the external circuit.

The results were gratifying in that, although the numerical values obtained were higher than those obtained under similar conditions experimentally, the relationship between breakdown voltage and recovery rate was predicted qualitatively throughout the range examined without the necessity of modifying the important details of the applied sparking mechanism. The breakdown voltage to be expected at a higher temperature was calculated for a particular voltage recovery rate and a lower value was obtained indicating that if it had been possible to use even higher temperatures in the calculations, numerical results more nearly in accord with observed results would have been obtained.

CHAPTER I

THE MECHANISM OF THE ELECTRIC SPARK

1. Theory of the spark.

Because of the many varied theories of spark discharge, the choice of a theory that would fulfill all the required conditions was a difficult one.

Since the classical Townsend theory of the mechanism of the electric spark requires such modification and has proven so unsatisfactory for the higher pressures and longer gaps encountered in circuit breaker work, an attempt was made to apply a recent theory known as the Streamer Theory which was developed by Loeb and Meek (8).

2. The streamer theory of spark discharge.

Let us consider a plane parallel gap of $\delta = 1$ centimeter in which the cathode is illuminated by ultra violet light to such an extent that one electron per microsecond leaves one square centimeter of cathode area. If the conventionally observed sparking potential, under these conditions, of $V_s = 31{,}600$ volts is impressed upon the electrodes, the field strength at any point in the gap will be $X = V_s/\delta = 31{,}600$ volts per centimeter since we may assume a uniform field across the gap.

An individual electron liberated from the cathode will start across the gap under the influence of the field and acquire a drift velocity v in the direction of the field. As it progresses, it releases new electrons by impact with neutral molecules at the rate of α per centimeter in the field direction.* At a distance x from the cathode, $\epsilon^{\alpha x}$ electrons will have been released forming what is known as an electron avalanche. Since the mobility of positive ions is so much smaller than that of the electron, $\epsilon^{\alpha x}$ positive ions will have been left behind virtually where they were formed by the detachment of an electron from a neutral molecule.

Since α is an extremely complicated function of the field strength and gas density, we must rely upon experimental values. The only data available in the range of field strengths and pressures with which we will be concerned are those of Sanders (12) who determined values of α at a constant temperature of 20 degrees centigrade for an $\underline{X}/p$ ratio between 20 and 160. As will be seen later, this upper value is still not high enough for our purpose. If we assume that a variation in temperature will effect α only in so far as it modifies the density of the gas, we may substitute $\underline{X}T/293p$ for $\underline{X}/p$ in Sanders results. With this modification, his values are plotted in figure (1).

As the electron avalanche advances, its tip is spread laterally by the random diffusion of the electrons.

* See appendix II.

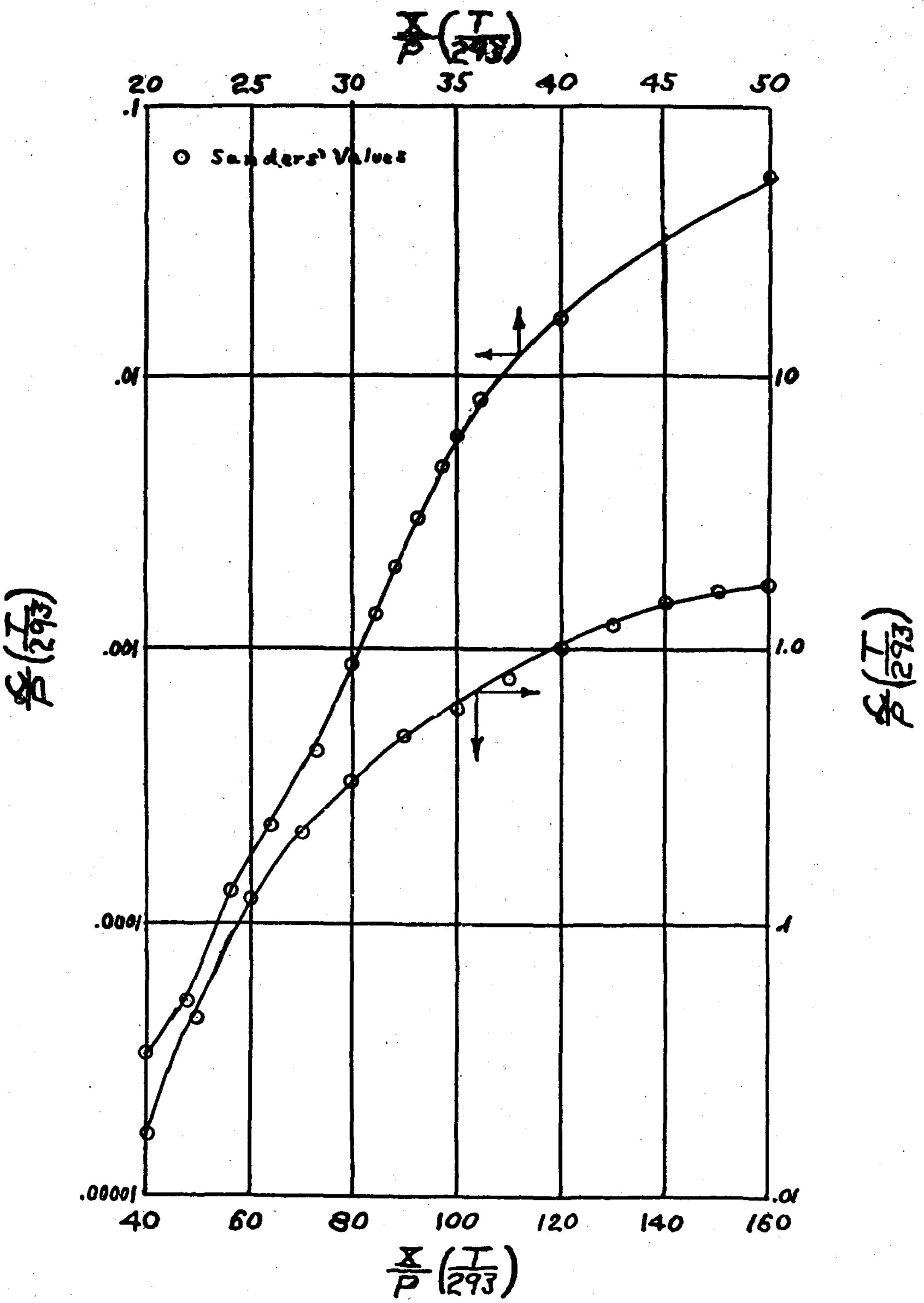


Fig. 1. - Values of the coefficient of ionization by collision in air versus the ratio of field strength to pressure with modification for temperature influence.

Most of the electrons in the avalanche will be drawn into the anode except for some few that are bound by the positive ions. The net result of these effects is shown schematically in figure (2). Such a distribution does not make a conducting filament of charges across the gap and therefore, in itself, does not constitute a breakdown of the gap.

The positive ions formed in this process could cross the gap and, by impact at the cathode, lead to an accelerated electron emission. However, due to their relatively slow movement, another mechanism may intervene and cause complete breakdown before this can occur.

As a result of the cumulative ionization, a positive ion space charge channel was left behind by the avalanche as was indicated in figure (2). Accompanying the ionization, there are from four to ten times as many excited atoms and molecules produced. These may emit short ultra violet radiation that is highly absorbed in the gas and leads to further ionization. The whole gas volume and the cathode are subjected to a shower of photons of all energies traveling with the velocity of light. Nearly instantaneously then, in the gap and at the cathode, new photo electrons are liberated which also begin to ionize cumulatively. The photo electrons created at any great distance from the space charge channel already formed will

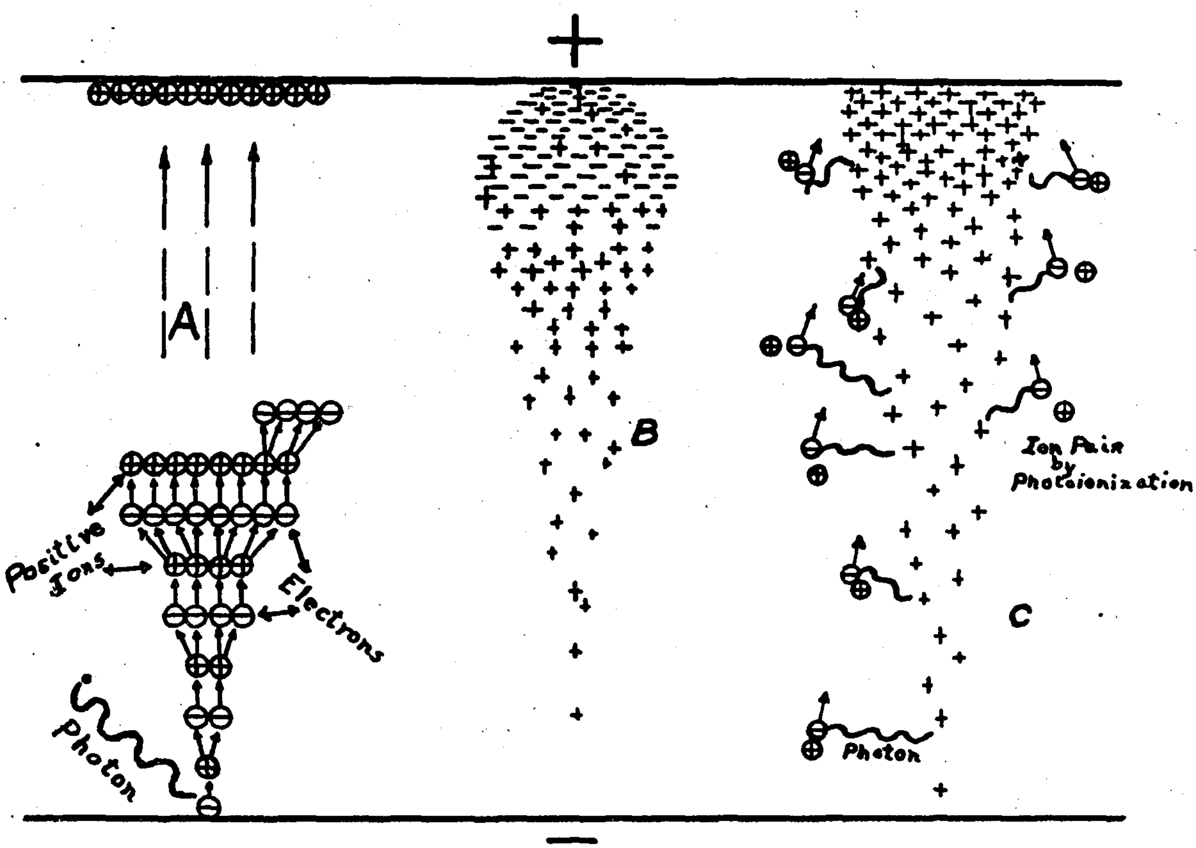


Fig. 2. - Schematic figures showing, A, the electron multiplication of electrons by cumulative ionization of a single electron liberated from the cathode by a photon; B, the avalanche has crossed the gap, spreading by diffusion; C, ion pairs out from the trail indicate the appearance of photoelectric ion pairs in the gas produced by photons from the avalanche.

be of no interest to us since they result only in later avalanches similar to the original and contribute nothing toward the breakdown of the gap. However, those photo electrons created near the space charge channel of positive ions, and especially near the very high concentration of ions near the anode, will be in a much enhanced field which will draw them into the space charge. If the space charge field is of appreciable magnitude, this action may be very effective since the increase in α due to the enhanced field may be quite large and produce intense cumulative ionization.

The electrons drawn to the positive space charge region feed into it creating a conducting plasma of electrons and ions. The positive ions left behind extend this high density positive space charge toward the cathode as a self propagating space charge streamer. Such streamers have been observed and photographed in cloud tracks by Raether (11). The manner of propagation is shown diagrammatically in figure (3). Their velocity of propagation, dependant upon photo ionization in the gas and photon propagation with the velocity of light, is probably much more rapid than the velocity of the initial electron avalanche.

As the streamer advances toward the cathode, it produces a filamentary region of intense space charge distortion along a line parallel to the field.

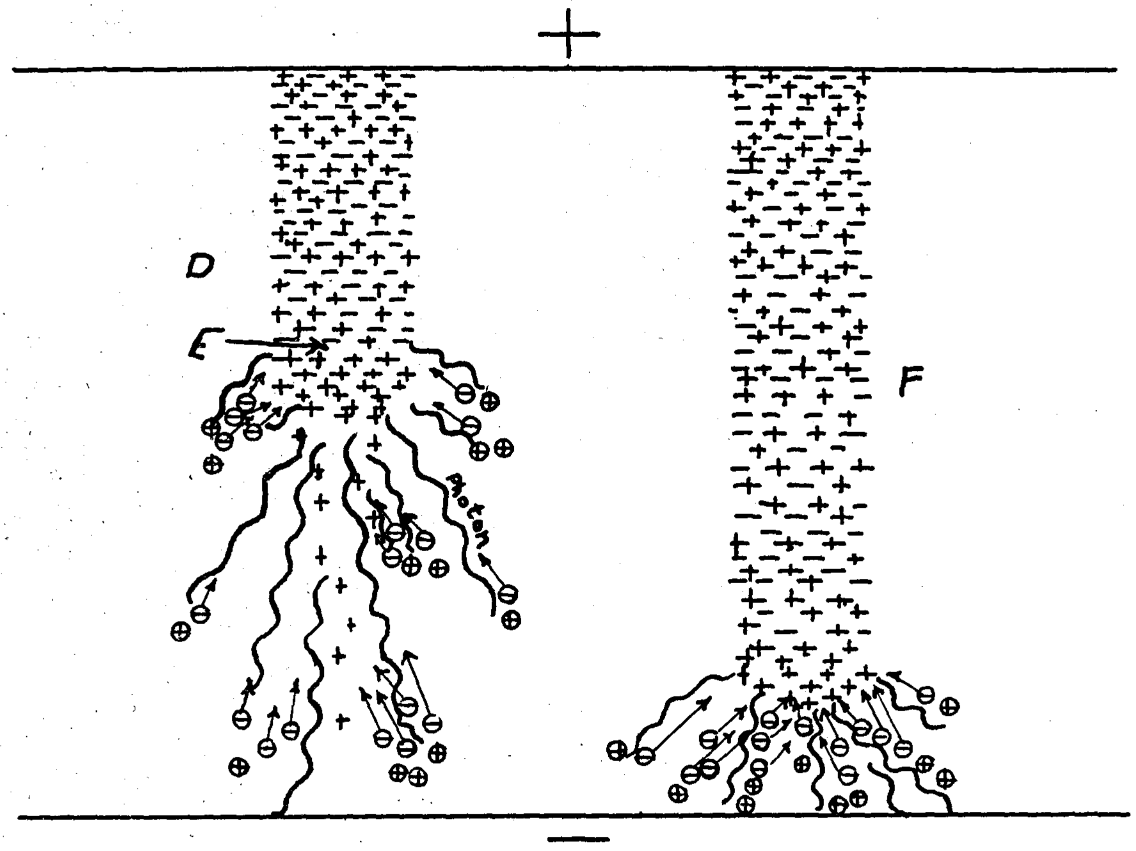


Fig. 3. - Schematic figure showing the growth of a streamer. At D, the channel of plasma has advanced one-half the way across the gap with its positive tip at E. The photo-ionization about the tip is shown with some ionization at the cathode. In F, the streamer approaches the cathode. The intense photo-ionization and photo-electric liberation from the cathode is indicated. Perhaps it is not emphasized sufficiently, for Raether's pictures show a strong cathode spot at this stage.

The potential distribution will be as shown in figure (4). The conducting streamer of plasma consisting of electrons and ions extending to the anode produces a very steep potential gradient at the cathode end of the streamer tip. As the tip reaches the cathode, the high field gives rise to intense ionization. A cathode spot forms and a rush of electrons from the cathode to the streamer tip results. A high potential wave passes up the pre-ionized conducting channel to the anode multiplying the electrons present by a large factor rendering the channel highly conducting. If the metal can emit a copius supply of electrons, the current will continue up the channel maintaining its high conductivity and even increasing it. Unless limited by external resistance, the current will develop into an arc.

It can be seen, then, that the radial field of the positive space charge residue left by an avalanche that had crossed the gap could be such as to draw photo electrons into itself and propagate as a positive anode streamer to the cathode. It has been shown that such a streamer, in crossing the gap, short circuited the electrodes by a conducting filament of plasma. It remains to be seen under what conditions such a streamer can occur and whether or not such conditions as are required do exist in a particular case.

First, an adequate density of photo ionization must

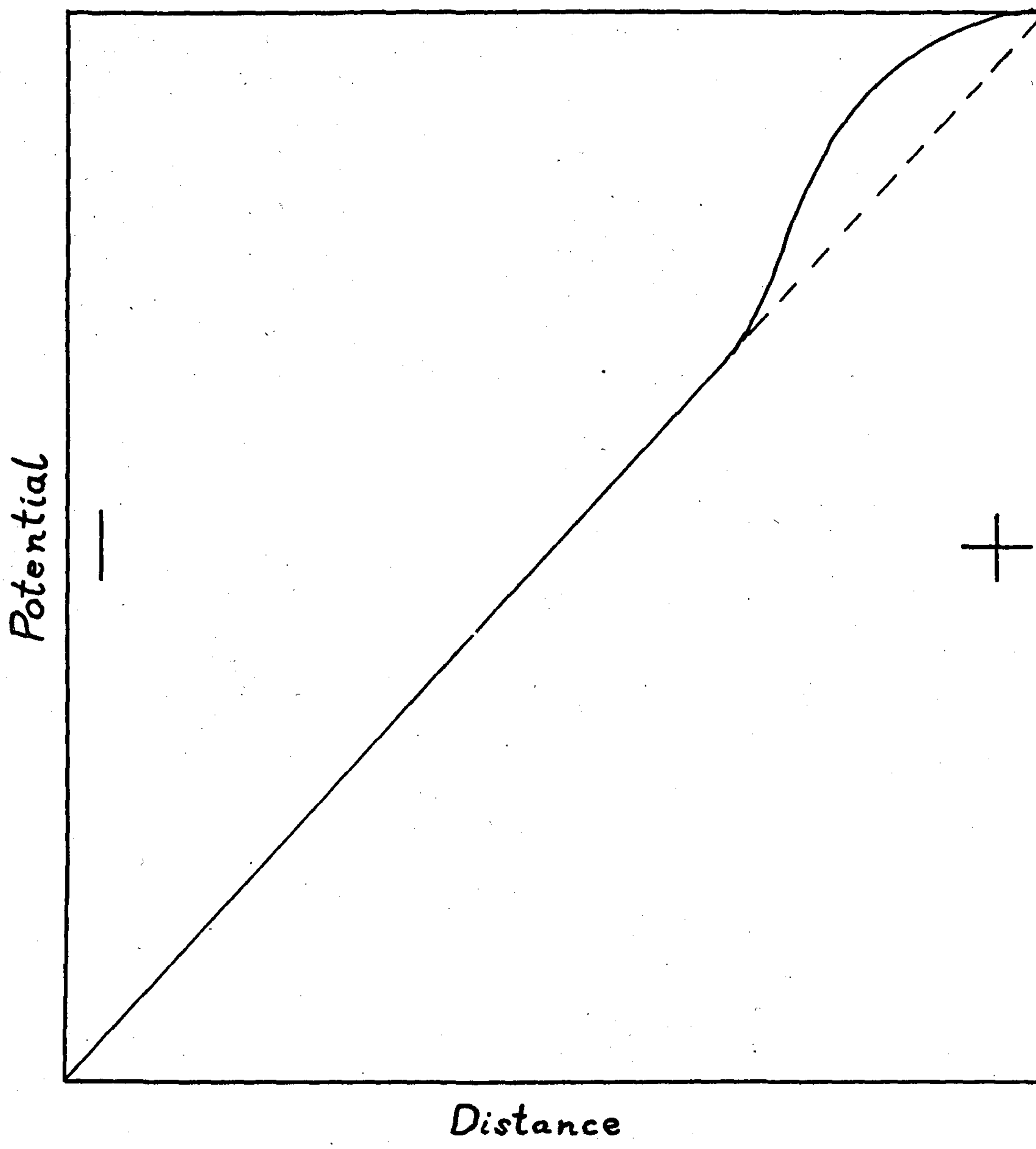


Fig. 4. - Space potential of streamer.

exist in the gas near the space charge to insure a continuous supply of electrons. This will depend on the diffusion of the electron avalanche (photon production in a small volume) and on the absorption coefficients of the molecules for the active photons produced in the avalanche.

Second, the positive space charge field must be of such intensity that the photo electrons produced are drawn into it and multiply sufficiently by cumulative ionization to cause the tip of the streamer to advance toward the cathode.

The question of the magnitude of the space charge field will be considered first. Meek (8) suggested that the ratio of the radial field strength to the impressed field might be the determing factor and proposed as a criterion that this ratio be between one tenth and one. With this in mind, let us return to the plane parallel gap with an electrode spacing of $\delta = 1$ centimeter. At a pressure of 760 millimeters of mercury and a temperature of 22 degrees centigrade, the conventional sparking potential, as previously mentioned is 31,600 volts and the ratio $\underline{X}T/293p = 41.6$. From the curve of figure (1), we get a value of $\alpha T/293p = .0224$ or $\alpha = 17$. As the electron avalanche proceeds across the gap, it leaves positive ions behind in its path. At a point distance x from the cathode, the rate of ion formation will be $\alpha \epsilon^{\alpha x}$ and the number of ions created in a distance dx at the end of this path will be $\alpha \epsilon^{\alpha x} dx$. As the electron avalanche advances, it is also diffusing outward at right angles to the field direction. While the ions are actually in a nearly conical channel with the area of maximum density at the anode end of the cone, it will be convenient for purposes of simplicity of calculation to consider them in a spherical volume of radius r equal to the average radial distance of diffusion. It can be seen by examination of the rate of ion formation that the density distribution will be such as to make any error introduced by such an assumption of small importance. The field strength $\underline{X}_1$ at the surface of this sphere will be $4\pi ne/4\pi r^2$ where n is the number of ions within the sphere and e is the electronic charge. The number of ions within the sphere must be equal to the ion density times the volume of the sphere so that n $= 4/3\, \pi r^3 N$ where N is the ion density so that the radial field strength may be expressed as $\underline{X}_1 = 4/3\, \pi r N e$. The density may also be expressed as the number of ions in a thin cylindrical volume of thickness dx divided by the volume of that cylinder which gives us

$$N = \frac{\alpha \epsilon^{\alpha x} dx}{\pi r^2 dx} = \frac{\alpha \epsilon^{\alpha x}}{\pi r^2} \qquad (1)$$

The field strength is then

$$\overline{X}_1 = \frac{4e\alpha\epsilon^{\alpha x}}{3r} \quad e.s.u. \tag{2}$$

Raether (11) gives the average radial distance of diffusion as $r = \sqrt{2Dx/v}$ but Loeb (8) indicates the value is more properly $\sqrt{4Dx/v}$. Since the drift velocity can be expressed in terms of the mobility and field strength, $v = k\overline{X}$, the expression can be modified to read $r = \sqrt{4Dx/k\overline{X}}$. The ratio D/k can be determined experimentally* and the expression then becomes

$$r = \left(.42x \frac{T}{273p}\right)^{1/2} \tag{3}$$

Combining equations (2) and (3) and converting from electro static to practical units gives for the radial field at the anode

$$\overline{X}_1 = \frac{2.96 \times 10^{-7} \alpha \epsilon^{\alpha x}}{\left(\frac{\delta}{p} \frac{T}{273}\right)^{1/2}} \quad volts/cm. \tag{4}$$

Substituting in the values previously mentioned for T, p, α, and δ gives $\overline{X}_1$ = 3200 volts per centimeter. Thus the radial field is approximately one tenth of the critical impressed field necessary to cause breakdown in this case.

* See Appendix 1.

With this evidence in mind, the authors of this paper chose as a criterion for breakdown of any gap the condition that an electron avalanche in crossing a gap would create at some point a space charge which would set up a radial field equal to one tenth of the impressed field at that point. While this may seem to be a very arbitrary and empirical solution, it must be remembered that this ratio has a definite physical significance based upon photon production, absorption coefficients for photons, and photo electric ionization but, due to our ignorance of the exact processes involved, theoretical evaluation is impossible.

Applying this criterion, we get for the "sparking threshold" of a plane parallel gap of length δ with a uniform impressed field the condition that the critical impressed field $\bar{X}_s$ that will just cause breakdown of the gap must be ten times the radial field $\bar{X}$, or

$$X_s = \frac{2.96 \times 10^{-6} \alpha \epsilon^{\alpha x}}{\left(\frac{\delta}{p} \frac{T}{273}\right)^{1/2}} \text{ volts/cm.} \qquad (5)$$

This equation must be modified somewhat for any distribution other than that of the uniform field. Since the field and thus the ratio $\bar{X}T/293p$ varies with x, α will also vary with x. To obtain the rate of ion formation at any point x, it is necessary to integrate $\alpha\,dx$ from zero to x. The general expression for breakdown with any field distribution then is

$$X_s = \frac{2.96 \times 10^{-6} \, \alpha_{x_s} \, \epsilon^{\int_0^{x_s} \alpha \, dx}}{\left(\frac{x_s}{p} \frac{T}{273} \right)^{\frac{1}{2}}} \text{ volts/cm.} \quad (6)$$

where x_s is the point where our criterion is satisfied with the critical breakdown voltage impressed and α_{x_s} is the value of α at point x_s .

So far we have established a criterion for satisfying only one condition, that of sufficient space charge field intensity. We have yet to establish a criterion by which we may judge whether an adequate density of photo ionization exists. The question resolves itself into whether there are conditions whereby the radial field could attain the required magnitude and yet the photo ionization be inadequate. Practically nothing is known about photon production and photo electric ionization in electron avalanches. However, experimental evidence indicates that there is a fairly close relationship between photon density and ion density within any given region of values of $\bar{X}T/293p$. If we assume that the ratio of the ion density to the density of photons which are photo electrically active in the gas will remain constant over a limited range, we may use the ion density as a criterion for estimating the density of photon production. We may then rephrase our question to whether the radial field can attain the required magnitude and yet have too low an ion density. From an inspection of equations (1), (2), and (3), it can be seen that, since the density is inversely proportional to the square of the radial distance of diffusion and the field strength inversely proportional to radial distance to the first power, such a possibility does exist.

Loeb (8) found experimentally that with gaps of the order of $\delta = 1$ centimeter, streamer formation is uncertain in a uniform field if the ratio $293p\,\delta/T$ was less than 200 due to inadequate photo ionization. We may then choose as a minimum value of ion density to insure streamer formation the value of the density under conditions when $293p\,\delta/T = 200$ and $\delta = 1$ centimeter. To find this density, we must solve equation (5) by trial and error to determine the value of $\overline{X}_s$ and the corresponding α that will satisfy the equation, and then substitute the value of α obtained into the equation for density.

If we introduce the above constants, equation (5) reduces to

$$X_s = 4.04 \times 10^{-5} \alpha \epsilon^{\alpha} \text{ volts/cm.} \tag{7}$$

or

$$\text{Log } X_s + 10.221 = \alpha + \log \alpha \tag{7a}$$

Introducing successive values of $\bar{X}_s$ and the corresponding values of α from figure (1) gives for the two sides of the equation

X_s		LEFT SIDE	RIGHT SIDE
10,000 volts/cm.		19.43134	13.48513
10,500	"	19.48013	15.88775
11,000	"	19.52665	19.72730

It can be seen that the correct values of $\bar{X}_s$ lies between 10,500 and 11,000 volts per centimeter. Further trials give a value of $\bar{X}_s$ = 10,965 volts per centimeter with a corresponding value of α = 16.71. From equations (1) and (3), the density is determined to be N = 4.2655×10^{10} .

By a modification of equation (1) similar to that used to develop equation (6) the density of ionization with any field distribution is

$$N = \frac{206.81\, \rho\, x_s\, \alpha_{x_s}\, \epsilon^{\int_0^{x_s} \alpha\, dx}}{T} \qquad (8)$$

If the density calculated by equation (8) is in excess of 4.2655×10^{10} adequate photo ionization exists.

This discussion would not be complete without some comment as to the possibility of some other mechanism causing breakdown if a positive streamer fails to materialize. If the density of photo ionization is inadequate for streamer formation or if the field distribution is such that adequate radial fields are not created by an electron avalanche, the positive ions may have time to reach the cathode and by ionization by impact on the cathode lead to an accelerated electron emission and eventual breakdown. A second possibility is that by virtue of a badly distorted field, a streamer might form in the gap but be unable to proceed across the gap because of such low field intensity at some point that the radial field would be attenuated by high rates of diffusion. This phenomena is most likely to occur when the streamer forms near the cathode, proceeds back to the cathode, and then attempts to advance again by successive avalanches and retrograde streamers through a region of low field intensity. Here the filamentary distortion caused by the radial field is in such a direction as to oppose the impressed field and further weaken it. Once again breakdown might occur by the action of positive ions at the cathode. Finally, if by some means, the impressed field is varied with time, the formation of a streamer might be delayed long enough for the positive ions to proceed to the cathode and cause breakdown. Both of these latter conditions are highly possible in the arc path of a circuit breaker at the time of current zero as will be seen and are extremely hard to identify except by experimental means because of the many variables which influence the two mechanisms.

CHAPTER II

INTERRUPTION OF AN ELECTRIC ARC

1. Theory of arc interruption.

With the theory of arc formation just described in mind, let us consider how it may be applied to the problem of interrupting the arc once it has been formed.

Since this discussion is limited to the quenching of arcs formed in the opening of a circuit carrying an alternating current, we may assume that at some time shortly after the contacts have been separated the arc current will be decreased to zero. The problem of extinguishing the arc then is reduced to that of preventing the re-establishment of the arc after that current zero. The extinction or reignition of the arc depends upon the outcome of a kind of race between two contending factors, one depending upon the external circuit and the other upon the space between the electrodes. The first is the rate at which the voltage applied to the electrodes by the external circuit builds up and the second is the rate at which the arc space recovers dielectric strength.

It has been shown that to establish an arc initially across a gap, a potential equal to or greater than a certain critical potential must be impressed across the gap. The field set up in the space between the electrodes causes an electron avalanche, originating at the cathode or from some point in the gap, to progress toward the anode. If the rate of diffusion of the electrons in the avalanche is sufficiently low, a positive space charge will be built up near the anode of such intensity that a positive streamer will propagate back to the cathode. When the streamer nears the cathode, the extremely high field intensity which results will cause a copius emission of electrons from the cathode. The electrons will pass up through the streamer to the anode making a highly conducting path across the gap. If the cathode can emit an adequate number of electrons, an arc will develop.

In the circuit breaker, however, we have the condition of an arc that is already bridging the gap if the unlikely case of very rapid opening of the contacts at exactly current zero is disregarded. As a result, there exists, between the contacts, a very highly conducting plasma of ions. Since the arc while it is playing takes a voltage which is generally smaller than the voltage generated in the circuit, the alternating current breaker depends for its operation upon the medium containing the arc returning from its condition of a comparatively good conductor, carrying current at a low voltage, to its normal condition of a comparatively good insulator withstanding the full generated voltage of the circuit with passage of insignificant currents. This transition, as mentioned, must be made before the circuit voltage recovers sufficiently to re-ignite the arc.

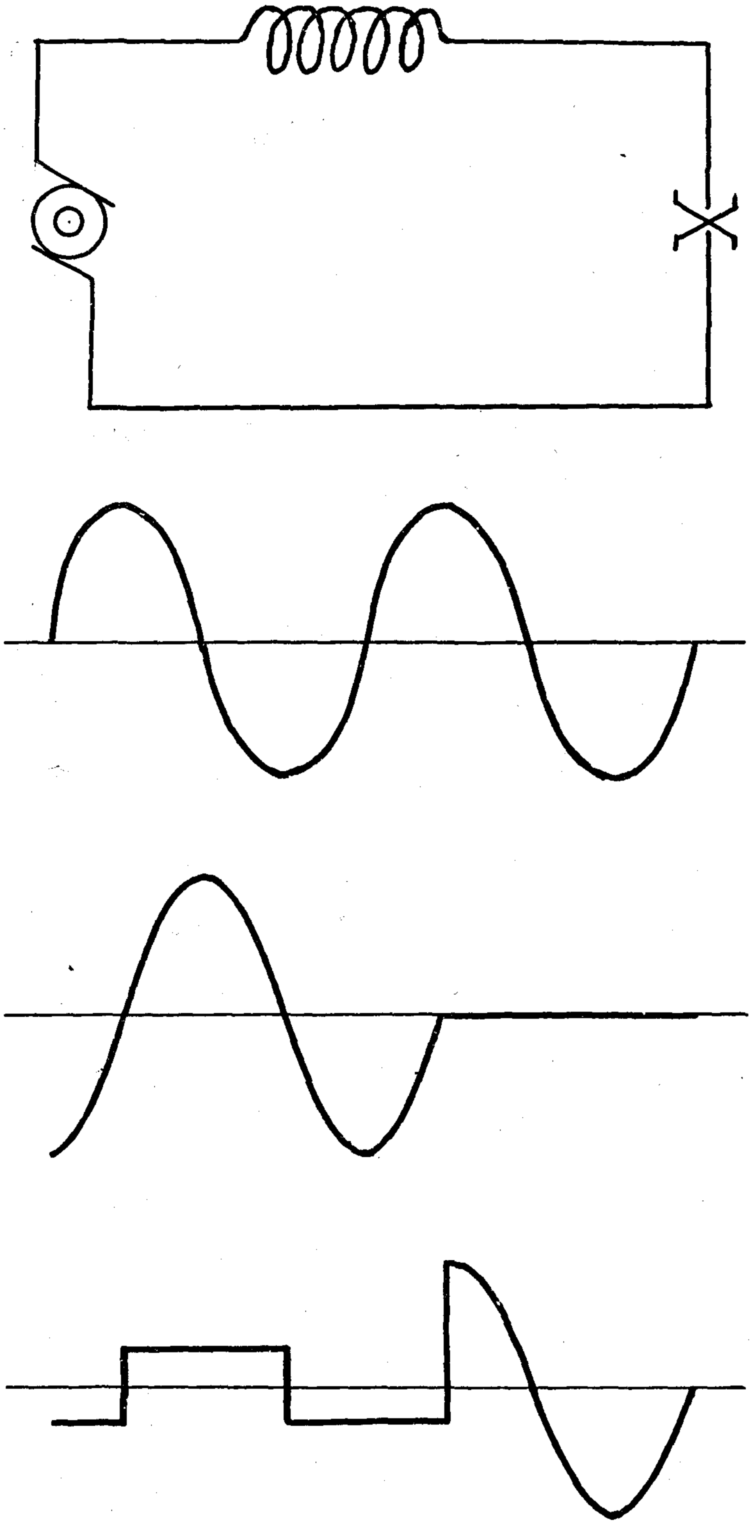
Since we have postulated breakdown by a streamer mechanism, we may assume that the ionization of the air between the electrodes will affect the breakdown voltage by the manner in which it distorts the field across the gap. If the conditions in the gap are such that the impressed voltage at no time is capable of setting up a field distribution conducive to streamer formation, the arc will not reignite. If, however, at any instant, the impressed voltage can create a field distribution such that a positive streamer will form, then, subject to some restrictions previously mentioned, breakdown will ensue. It seems apparent that, to determine whether or not the arc will reignite, we must first calculate the voltage impressed on the electrodes at any instant and then determine the field distribution existing at that instant. With this information, we can turn to the equations for the sparking "threshold" to determine if streamer formation is probable.

2. Recovery of the external circuit.

The transition from conducting medium to dielectric after the current decreases to zero in its normal cycle cannot take place instantly. Since the conductivety of the arc is due to a dense plasma of electrons and ions, time must be allowed for these ions to disappear or at least assume a new orientation if reignition is to be prevented. The time available for this transformation must be determined.

Consider first a simple circuit consisting of an alternating current generator in series with an ideal reactor and an arc, as in figure (5-a). Assuming the arc voltage is small compared to the generator voltage, the current will lag the voltage by nearly 90 degrees as shown in figure (5-c) while the arc voltage will be in phase with the current, figure (5-d). If at the end of a half cycle of current, the arc should extinguish and the current remain zero, the voltage across the electrodes would immediately rise to the full instantaneous value of generated voltage, which, because of the phase relationship between current and generator voltage, would be almost the peak value. In such a circuit, it is obvious that no time would be allowed for the arc path to lose its conductivety after the current zero. When the arc extinguished, the gap between the electrodes would be subjected to full generator voltage and, since we assumed the resistance of the arc to be comparatively low, the arc would reignite.

However, an actual reactor may be thought of as an ideal reactor shunted by a small condensor as in figure (6-a). In such a circuit, the voltage across the arc terminals will not rise instantly to generator voltage but will approach it gradually as the circuit oscillates as shown



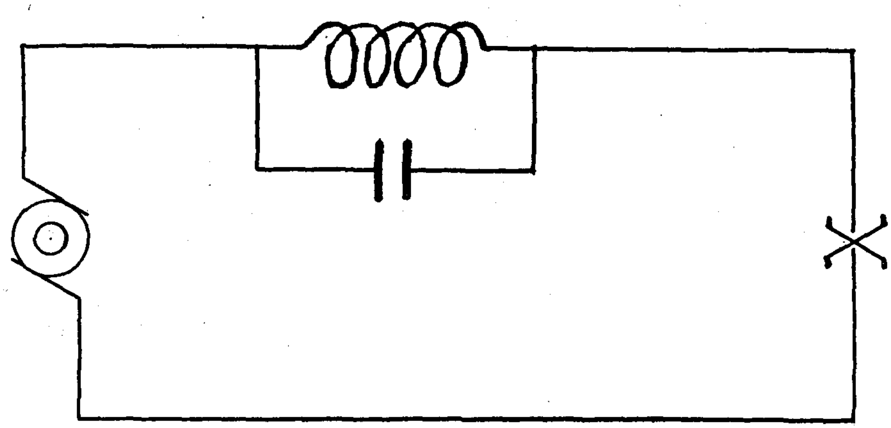
a) Circuit with ideal reactor.

b) Generated voltage.

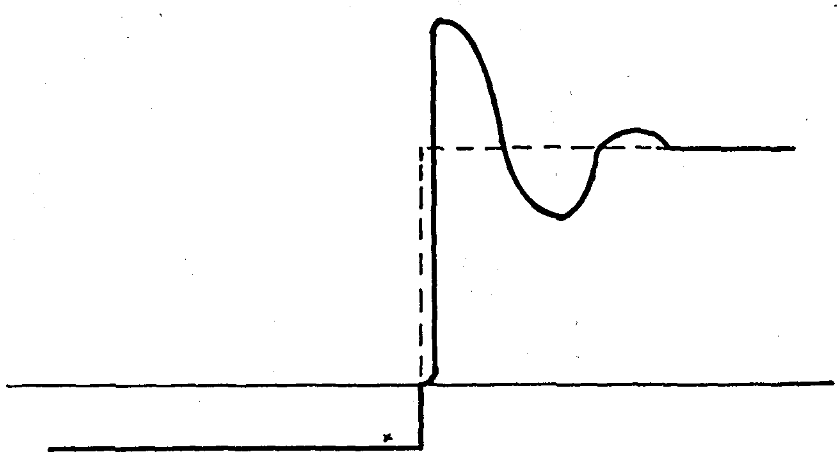
c) Current.

d) Arc voltage.

Figure 5.



a) Circuit with actual reactor.



b) Arc voltage.

Figure 6.

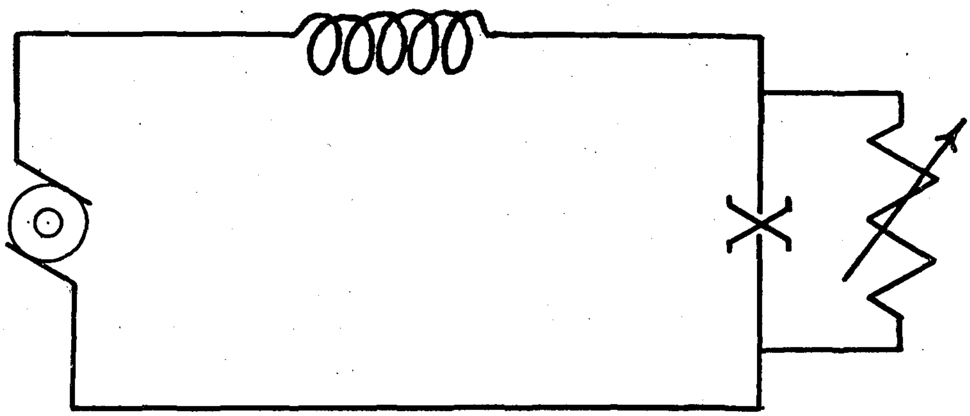
in figure (6-b). The time for the voltage impressed on the arc terminals to reach its maximum is seen to be one quarter of the period of a natural oscillation of the reactor and this is the time available for the transition from conducting plasma to insulating air space.

Generalizing, the time available for this transition is at least one quarter of a period of free oscillation of the circuit external to the arc. In a practical power circuit, this time varies from 2.5 micro seconds for a current limiting reactor with natural frequency in the order of 100,000 cycles per second to several thousand micro seconds for a long transmission line with a natural frequency of a few hundred cycles per second.

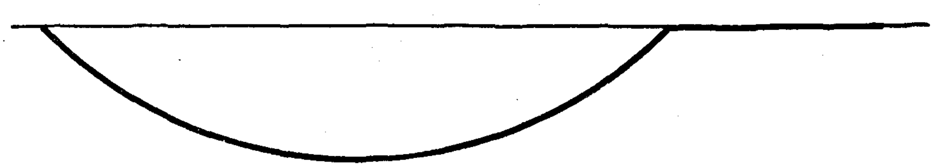
The nature of the external circuit, since it determines the time available for transition, will greatly affect the interrupting capacity of an alternating current breaker.

The methods of controlling the recovery rate of the voltage across the terminals in a practical circuit will not be discussed here but the effect of one variation of the circuit used in experimental work will be demonstrated.

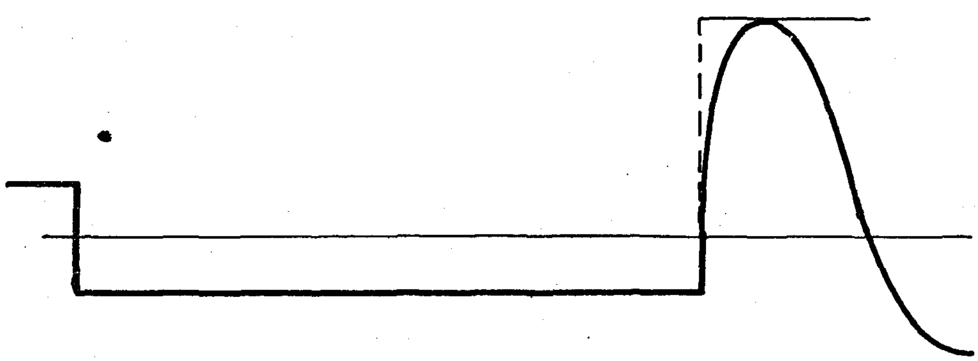
The arc is shunted by an adjustable resistance as in figure (7-a). Since we are interested in events occurring in at most several thousand micro seconds after the current zero, the generator voltage may be considered constant and in this circuit approximately equal to the peak voltage.



a) Circuit with shunted arc.



b) Arc current.



c) Arc voltage.

Figure 7.

Conditions in the circuit immediately following the extinction of the arc can be expressed by the differential equation

$$\frac{di}{dt} = \frac{R}{L} i = \frac{V_{max}}{L} \tag{9}$$

which by conventional mathematics gives the voltage across the arc terminals as

$$V = V_{max} \left(1 - \epsilon^{-\frac{R}{L}t} \right) \tag{10}$$

where V_{max} is the peak voltage. The rate of voltage rise will be

$$\frac{dV}{dt} = \frac{V_{max}}{L} R \epsilon^{-\frac{R}{L}t} \tag{11}$$

and the maximum rate of rise which comes immediately after the current zero is

$$\left(\frac{dV}{dt} \right)_{max} = \frac{V_{max} R}{L} \tag{12}$$

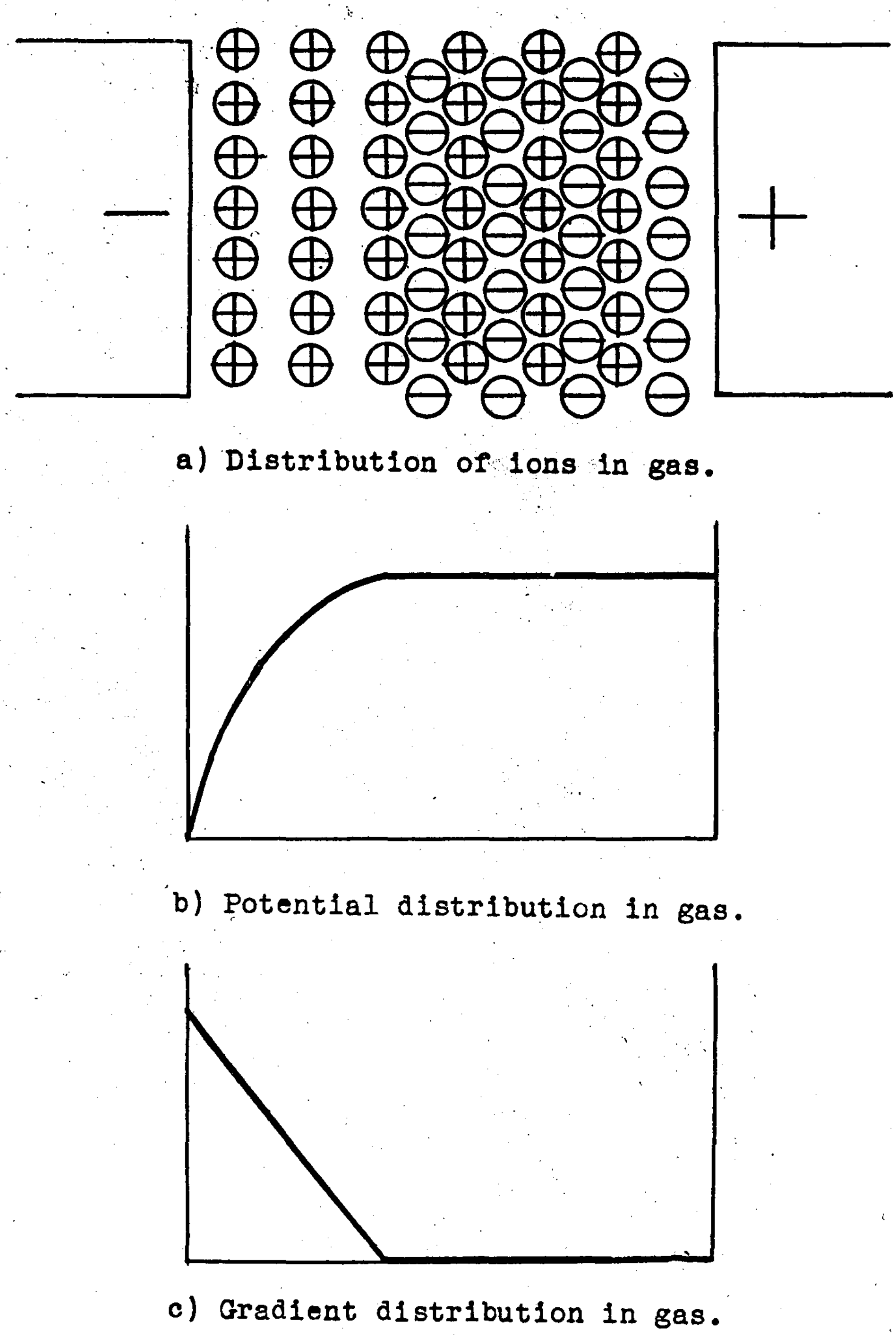
Changing the value of the shunting resistance will thus vary the rate of voltage recovery. This initial rate of voltage rise will be used later as a limiting case to determine how rapidly the arc space must recover dielectric strength if reignition is to be prevented.

3. Deionization of the arc space.

Before attempting to apply the theory of the spark to the problem of breakdown, it will be necessary to consider at some length the phenomena taking place in the gap just before and immediately following the current zero.

As the voltage impressed on the arc terminals by the external circuit increases, it acts upon a gas space in which the density of ionization is decreasing. What, then, is the relationship between dielectric strength and ion density in a gas.

Just prior to the current zero, we may assume a uniform ion density across the gap due to the highly conducting plasma previously described. However, as the arc voltage builds up, this distribution is distorted by the electric field. At the cathode, negative ions are repelled and positive ions are attracted, while at the anode, a similar, although reversed, action takes place. Since the mobility of the positive ions is very small compared to that of the negative ions, a positive space charge will develop next to the cathode which will increase the electrical gradient there. The resulting ion distribution with consequent potential and gradient distributions are depicted in figure (8). This space charge, built up by the movement of negative ions away from the cathode will cause a considerable portion of the impressed voltage to be consumed



a) Distribution of ions in gas.

b) Potential distribution in gas.

c) Gradient distribution in gas.

Figure 8.

in the region next to the cathode. As its thickness increases, practically all the impressed voltage will be consumed by this cathode space.

Taking the case of an arc in the open and assuming that the diameter of the ionized arc path is large compared with the gap length, let us consider the space next to the cathode containing only positive ions.

Let L = thickness of cathode space at time t

i = current density at time t.

V = potential at a point x distance from the cathode at time t

X = field intensity at point x, $= \partial V / \partial x$

k = mobility of positive ions

v = velocity of positive ions toward cathode at point x

e = charge on ion = 4.803×10^{-10} e.s.u.

N = density of positive ions in cathode region at point x

Because of our assumption of large diameter of arc path compared to short gap length, we may apply Poissons equation for one dimension which gives

$$\frac{\partial^2 V}{\partial x^2} = -4\pi N e \qquad (13)$$

Also, the current density, neglecting the displacement current, $\frac{1}{4\pi}\frac{\partial X}{\partial t}$. will be

$$i = Nev \tag{14}$$

and

$$v = kX = k\frac{\partial V}{\partial x} \tag{15}$$

Eliminating N and v, we arrive at the equation

$$\frac{\partial^2 V}{\partial x^2} \cdot \frac{\partial V}{\partial x} = -\frac{4\pi i}{k} \tag{16}$$

Integration gives

$$\frac{\partial V}{\partial x} = \sqrt{-\frac{8\pi i x}{k} + A} \tag{17}$$

Substituting equation (17) into equation (13) gives

$$N = \frac{i/e}{k\sqrt{-\frac{8\pi i x}{k} + A}} \tag{18}$$

Turning to the main body of the gas, let

N_0 = density of positive ions in body of gas at time t

N_{00} = initial density of positive ions

α_r = coefficient of recombination of ions

B = rate of increase of impressed voltage

V_0 = Bt = potential impressed on electrodes assuming linear increase in voltage with time

If we assume that deionization of an arc in the open takes place primarily by recombination and if we further assume that the densities of positive and negative ions in the main body of the gas remote from the cathode space are approximately equal, then the rate of deionization of the main body of the gas is given by

$$\frac{dN_o}{dt} = -\alpha_r N_o^2 \tag{19}$$

which when integrated gives

$$\frac{1}{N_o} = \frac{1}{N_{oo}} + \alpha_r t \tag{20}$$

Since from our theory of the arc N_{oo} is very large, we may simplify by considering $1/N_{oo}$ negligible giving

$$N_o = \frac{1}{\alpha_r t} \tag{21}$$

At point x = L, N must equal N_o , hence,

$$N_o(L) = N(L) = \frac{i/e}{k\sqrt{-\frac{8\pi i}{k}L + A}} \tag{22}$$

Therefore,

$$A = \left(\frac{i}{ekN_o}\right)^2 + \frac{8\pi i L}{k} \tag{22}$$

Near the boundary of the space charge, fresh charge is continuously being exposed by the resultant motion of the positive and negative ions at the rate of

$$\frac{dL}{dt} = \frac{i}{N_o e} \tag{23}$$

Finally, if the gradient in the body of the gas is negligible

$$V_o = V(L) \tag{24}$$

while at x = 0

$$V = 0 \tag{25}$$

Substituting equation (22) into equation (17) gives

$$\frac{dV}{dX} = X = \left[\frac{8\pi i (L-X)}{k} + \left(\frac{i}{e k N_o} \right)^2 \right]^{1/2} \tag{26}$$

Integration of equation (26) and introduction of boundary conditions of equation (24) and (25) leads to

$$\beta t = V_o = \frac{k}{12\pi i} \left\{ \left[\frac{8\pi i}{k} L + \left(\frac{i}{e k N_o} \right)^2 \right]^{3/2} - \left(\frac{i}{e k N_o} \right)^3 \right\} \tag{27}$$

which with equation (21) and (23) form a complete system of equations of which the solution is

$$L = \frac{\sqrt{\beta}\, t}{\left(\frac{\alpha_r}{12\pi e k^2} \right)^{\frac{1}{2}} \left[\left(1 + \frac{8\pi e k}{\alpha_r} \right)^{3/2} - 1 \right]^{\frac{1}{2}}} \tag{28}$$

From equation (26), when x = 0, dV/dx is a maximum and is

$$\left(\frac{dV}{dx}\right)_0 = X_0 = \frac{L}{kt}\left(1 + \frac{8\pi e k}{\alpha_r}\right)^{1/2} \tag{29}$$

Substituting equation (29) back into equation (26) gives for the gradient at any point x

$$X = X_0\sqrt{1 - \left(\frac{\frac{8\pi e k}{\alpha_r}}{1 + \frac{8\pi e k}{\alpha_r}}\right)\frac{x}{L}} \tag{30}$$

Introduction of appropriate values of the mobility of positive ions and the coefficient of recombination inequation (28) will give the distance of the boundary layer from the cathode in terms of the rate of recovery of the impressed voltage and the time elapsed since the current zero. With this value of L determined, equation (29) can be evaluated in terms of the recovery rate. With X_0 known for a given value of B, the field intensity X at any value of the ratio x/L can be calculated.

All the necessary information is now available and we may proceed to apply our theory of spark discharge.

4. Calculation of the breakdown voltage.

The generalized equation for breakdown was seen to be

$$X_s = \frac{2.96 \times 10^{-6} \alpha_{x_s} \epsilon^{\int_0^{x_s} \alpha \, dx}}{\left(\frac{x_s}{p} \frac{T}{273} \right)^{\frac{1}{2}}} \quad \text{volts/cm.} \tag{6}$$

To solve this equation, we must introduce the values of field intensity determined in the previous section but, since α will vary with X, the solution can only be obtained by trial and error. Also, since the field intensity is given in terms of x/L, we must introduce this ratio into equation (6) which then becomes

$$X_s = \frac{2.96 \times 10^{-6} \alpha_{x_s} \epsilon^{L \int_0^{x_s/L} \alpha \, d(x/L)}}{\left(\frac{x_s}{L}\right)^{\frac{1}{2}} L^{\frac{1}{2}} \left(\frac{T}{273p}\right)^{\frac{1}{2}}} \tag{31}$$

This equation is, however, very awkward to handle and for simplicity of calculation only is rearranged to read

$$L\int_0^{x_s/L} \alpha \, d\left(\tfrac{x}{L}\right) - \tfrac{1}{2}\log L = \mathrm{Log}\; 1.6691 \times 10^{-7}\left(\tfrac{T}{p}\right)^{\frac{1}{2}} - \log\frac{\alpha_{x_s}}{100} + \tfrac{1}{2}\log\frac{x_s}{L} + \log\frac{X_s T}{29300p} \tag{32}$$

If the temperature and pressure existing in the gap along the ionized path are introduced into equation (32), it can be seen that L will be a function of x/L alone since for a given value of x/L, the field intensity and coefficient of ionization by electron impact are fixed.

Only the minimum value of L so determined has physical significance since this value will give the minimum voltage that will cause breakdown as can be seen from an inspection of equations (27) and (28).

The minimum voltage was calculated for a number of different values of recovery rate and a definite relationship between this rate and the breakdown voltage was observed.

5. Breakdown voltage as a function of recovery rate.*

Since the values of the coefficient of ionization by electron impact were determined for a maximum value of XT/293p of 160, it was necessary to assume a temperature in the ionized path of 1,000 degrees centigrade in order to determine the breakdown voltage over a sufficiently large range of voltage recovery rates. The use of a higher temperature which would perhaps have been more in accord with physical facts gave values of the ratio in excess of 160.

At temperature of 1,000 degrees centigrade, the mobility is 876 centimeter per second per statvolt per centimeter and the coefficient of recombination is 3.135 cubic centimeters per second.**

* Tabulated results of individual calculations are given in Appendix III.

**See Apendices I and II.

With these values, equations (28) and (29) reduced to

$$L = .4873 \sqrt{B}\ t \tag{33}$$

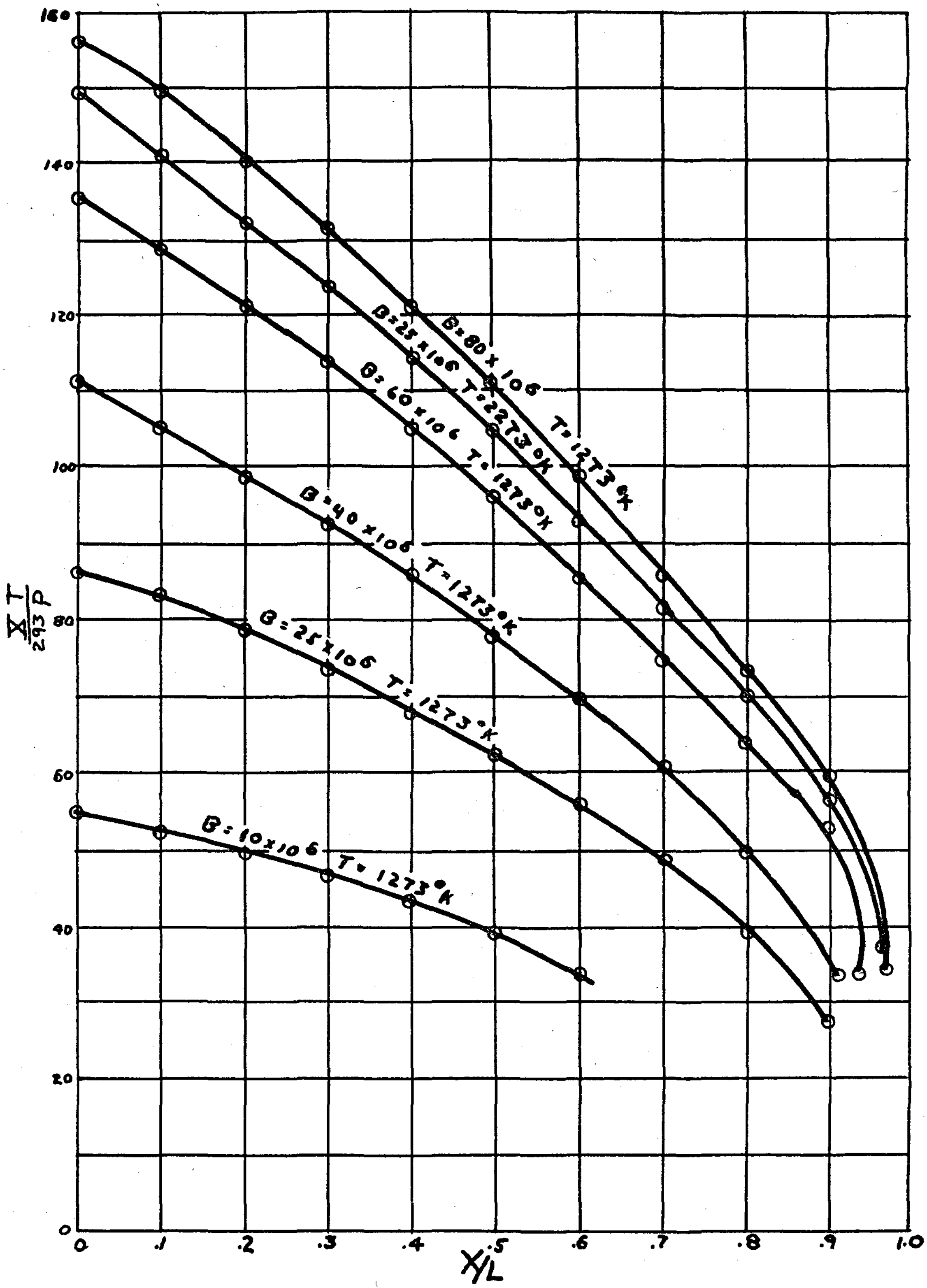
and

$$X_o = 3.096 \sqrt{B} \tag{34}$$

The values of X_o were calculated for several recovery rates and by substitution in equation (30) the field intensity at various points between the cathode and space charge boundary were found. From these values, the ratio XT/293p was computed and the variation across the gap is shown in figure (9). The values of α were then taken from the curve of figure (1) and these are plotted in figure (10). Corresponding values of α and X were then substituted in equation (32) and values of L were found which are plotted in figure (11). The minimum value of L is found by inspection of the curve and by using the relationship of equation (33) the breakdown voltage is determined.

The values of the breakdown voltage versus the voltage recovery rate of the external circuit are plotted in figure (12)

The only remaining problem was to determine whether the ion density at the end of the avalanche was sufficient to cause adequate photo ionization. The density was computed by use of equation (8) and the values are tabulated


B=80x106 T=1273°K
B=25x106 T=2273°K
B=60x106 T=1273°K
B=40x106 T=1273°K
B=25x106 T=1273°K
B=10x106 T=1273°K
160
140
120
100
80
60
40
20
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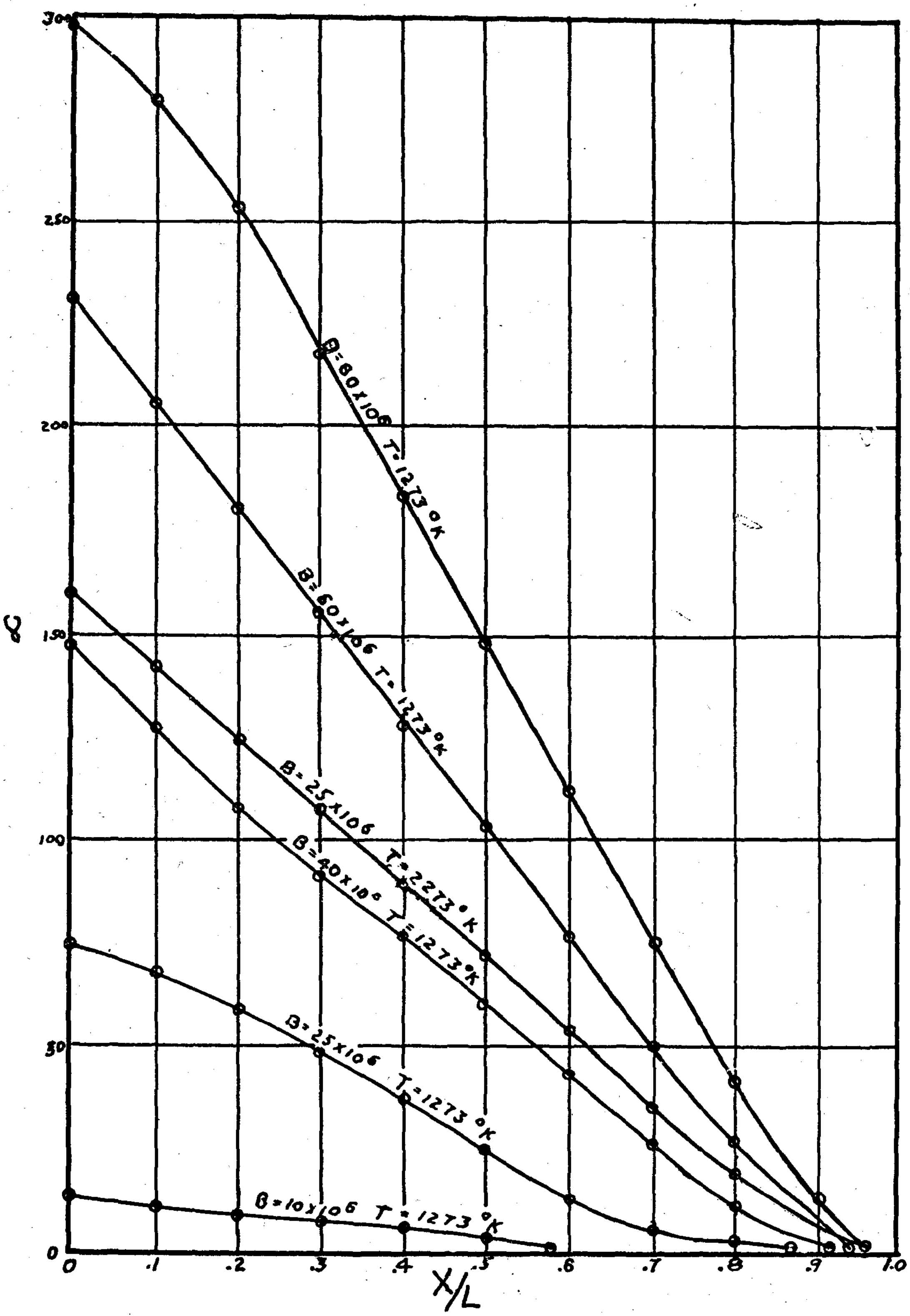
.6
.7
.8
.9
1.0
X/L

Figure 9.



300
250
200
150
100
50
0
C
0
.1
.2
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.4
.5
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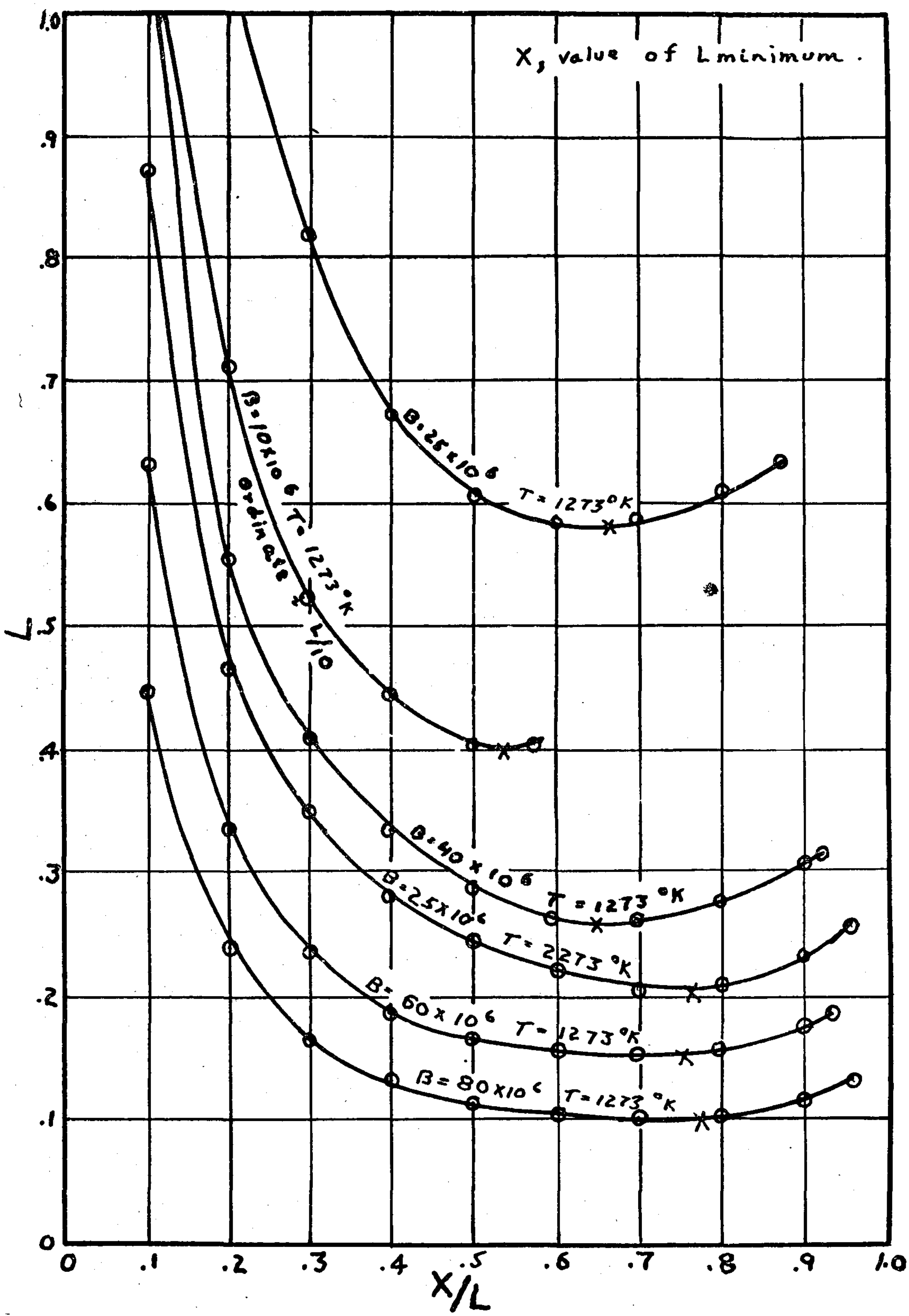
1.0
X/L
B=80x10⁶ T=1273 °K
B=60x10⁶ T=1273 °K
B=25x10⁶ T=2273 °K
B=40x10⁶ T=1273 °K
B=25x10⁶ T=1273 °K
B=10x10⁶ T=1273 °K

Figure 10.


X, value of L minimum.
B=25×10⁶
T=1273°K
B=10×10⁶ T=1273°K
ordinate = L/10
B=40×10⁶ T=1273°K
B=25×10⁶ T=2273°K
B=60×10⁶ T=1273°K
B=80×10⁶ T=1273°K
L
X/L
0
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.8
.9
1.0

Figure 11.

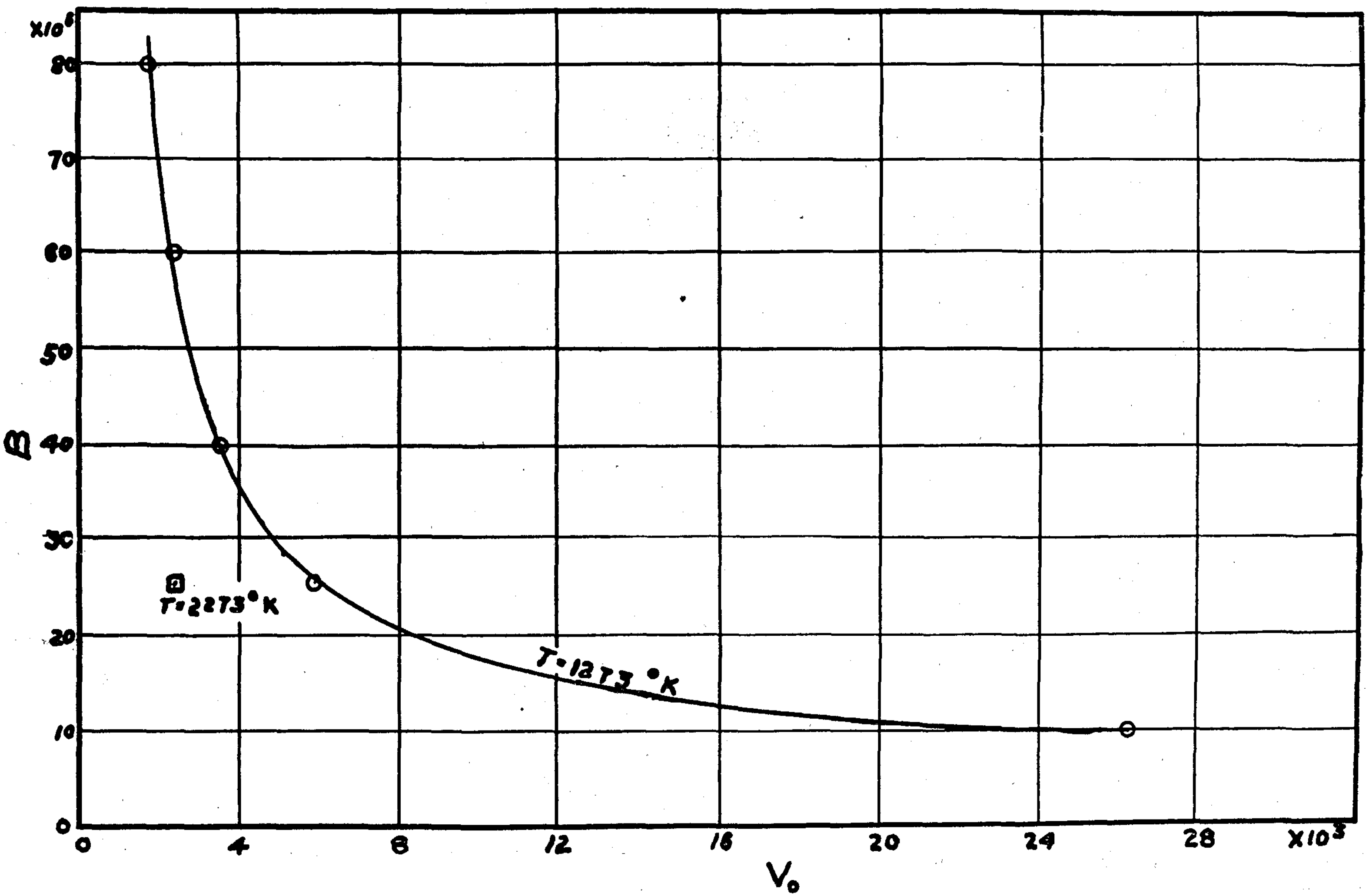

Fig. 12. - Computed values of recovery of dielectric strength of short arcs in air.

along with other results in figure (17) of Appendix III. As can be seen, all values were above the minimum required value of 4.2655×10^{10} so that in all cases the density is sufficient for streamer formation.

In order to find the possible effect of temperature change, one calculation was made for a temperature of 2,000 degrees centigrade. The results obtained are tabulated and plotted with the values of 1,000 degrees and the marked lowering of breakdown voltage should be noted.

CHAPTER III

THE APPLICABILITY OF THE STREAMER THEORY

1. Comparison of results with observed phenomena.

It can be seen by inspection of the results tabulated in figure (17) of Appendix III that the breakdown voltage, as predicted by our sparking theory, follows a definite pattern with respect to the rate of recovery of the voltage impressed on the terminals by the external circuit.

At low values of the voltage recovery rate, a small increase in the rate causes a considerable change in the voltage required to cause breakdown. As the recovery rate increases, however, this effect is much less pronounced until finally at very high rates of recovery, the breakdown voltage tends toward a constant value. The results are plotted in figure (12) and the general shape of the curve should be noted for it is upon this shape that the usefulness of the streamer theory in predicting the action of a circuit breaker will be judged.

Experimental determinations of the manner of variation of breakdown voltage with variable rates of voltage recovery are very limited. In one series of experiments of this type, Slepian (13) found the voltage required to cause reignition of an arc between plates spaced one sixteenth of an inch apart. By using a circuit similar to that of figure (7), he could, by adjusting the value of the shunting resistance, vary the initial rate of rise of the voltage. To avoid contamination of the arc space by metallic vapors, an auxiliary magnetic field was impressed which moved the arc over the surface of the electrode at such a rate that melting or vaporization of the electrode prior to the extinction of the arc did not occur. The results obtained are plotted in figure (13).

A comparison of this curve with that obtained by the authors from their theoretical analysis shows a certain qualitative agreement as to the manner of variation. However, the numerical results are widely different.

Before attempting to evaluate the results of this investigation, it will be well to consider carefully the differences between the conditions assumed and those of the actual experiment. In the theoretical analysis, a temperature of 1,000 degrees centigrade was assumed, except for one calculation at 2,000 degrees centigrade, for reasons which were explained in the calculations. The authors believe that this temperature is well below the actual value. Secondly, a constant rate of voltage rise was assumed, where as, the voltage rise actually used in the experiments varied in accordance with equation (11) and as indicated in figure (7). Finally, the gap length in the experiments was one sixteenth of an inch, while in the analysis the only restriction placed on gap length was that the electrode

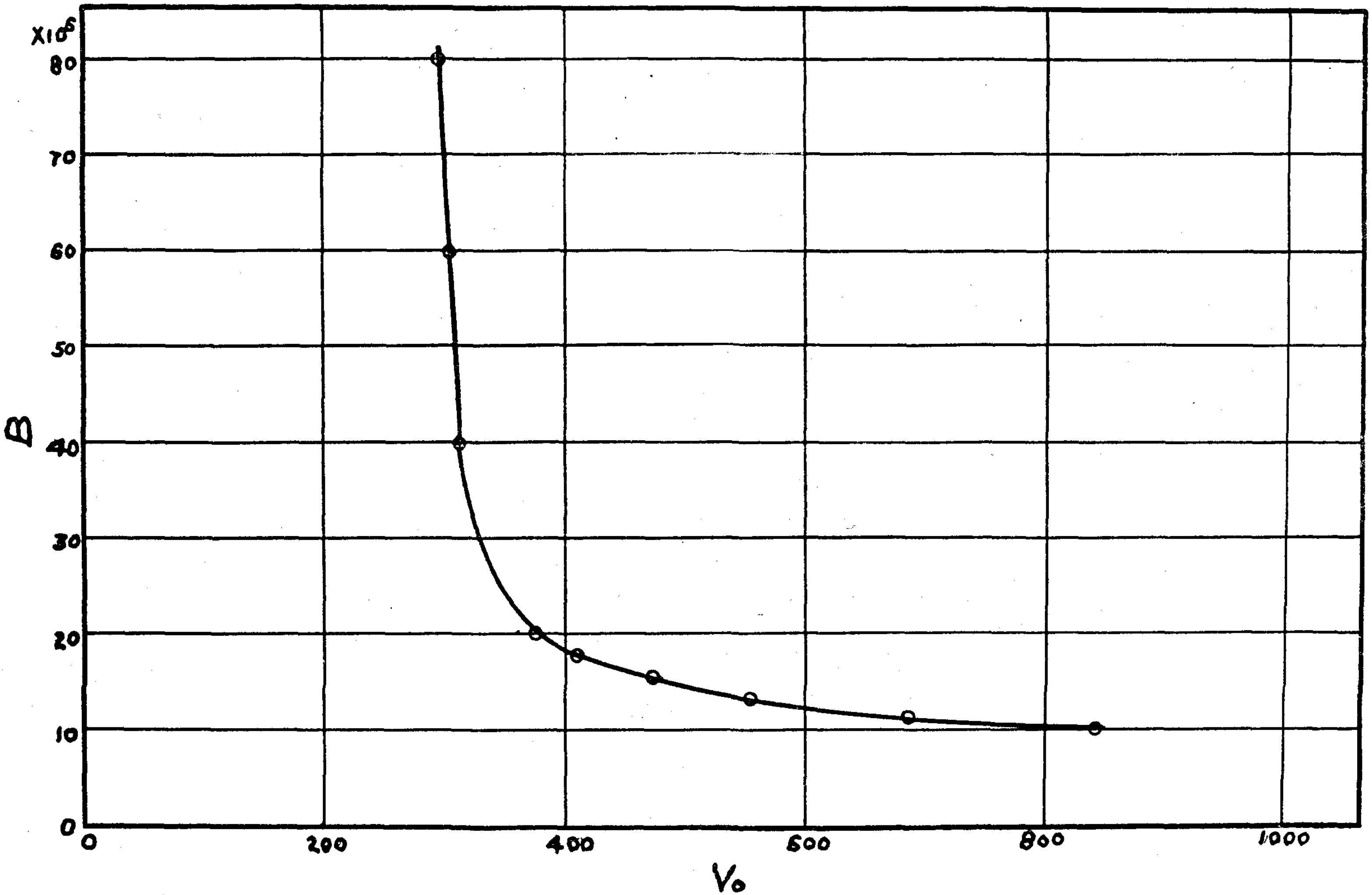

Fig. 13. - Experimentally determined values of recovery of dielectric strength of short arcs in air.

spacing be short so as to avoid, as much as possible, introducing complications in the manner of deionization on surfaces and excessive diffusion and cooling of the ionized plasma between the cathode space charge boundary layer and the anode.

2. A critical analysis of the results.

From the results obtained in this analysis, the authors have concluded that the streamer theory of spark discharge can be of considerable value in predicting the efficacy of a circuit breaker in interrupting the current in a circuit under a given set of conditions.

Since the numerical results were so widely at variance with the observed values, this statement requires considerable explanation.

The marked lowering of the breakdown voltage for a recovery rate of 25×10^6 volts per second, when 2,000 degrees was assumed as the temperature in the ionized path immediately after the current zero indicates that more definite information as to this temperature is necessary before the theory can be expected to give results which are quantitatively correct. To utilize this temperature, when determined, in the calculations, accurate determinations of the mobilities and coefficients of recombination at these temperatures must also be made or at least the variation with temperature at higher temperatures must be known. In addition, the range of values of the coefficient of ionization by electron impact must be extended to give values under conditions where the ratio XT/293p is greater than 160.

The possibility still exists, however, that breakdown does not occur by a streamer mechanism. The elapsed time between the current zero and the predicted breakdown as determined is probably sufficient for positive ions to move to the cathode and cause a accelerated electron emission and eventual breakdown. Since the time is a function of all the variables whose values are considered doubtful, as can be seen from equations (28) and (33), this possibility must await further verification. Also, the streamer once formed, might be unable to extend itself into the weak field region near the boundary layer of the cathode space charge by succeeding avalanches but this possibility can only be ruled out by comparison of future results with observed data.

The authors of this paper, however, are basing their belief in the essential applicability of the streamer theory to circuit breaker problems upon the qualitative agreement as to the manner of variation of breakdown voltage with the recovery rate of the voltage impressed by the external circuit as shown by the curves of figures (12) and (13). It is believed that if the theory were in error to the extent that breakdown actually occurred by some other mechanism, no such agreement would be probable.

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APPENDIX I*

THE DETERMINATION OF MOBILITIES AND COEFFICIENTS OF DIFFUSION

1. Motions of electrons and ions.

If a charged particle is placed in an electric field, it will experience an acceleration due to the action of the field. If, however, the particle is immersed in a gas, the acceleration cannot continue as the particle will soon collide with a neutral molecule. If a large number of such impacts occur in a short distance of travel, the motion may be reduced to a uniform drift in the field direction. The velocity will be a function of the field strength although not in all cases directly proportional. It is convenient, however, to express this relationship as if it were a direct proportionality and to call the proportionality constant the "mobility", with the realization that the mobility is not truly a constant but may actually be a function of field strength itself. Obviously, the mobility as just defined must largely depend upon the increase in velocity between impacts and hence upon the distance of travel between impacts.

In addition to the motion of the charged particles

* References - (2), (4), (6), (7) and (14).

in the direction of the field, there is another process known as diffusion. The rate of diffusion of charged particles in a gas will be proportional to the concentration gradient of the particles in the gas and we define the proportionality constant as the "coefficient of diffusion".

The motions of electrons and ions in a gas are seen to be in general functions of the field strength, the mass of the particle, charge on the particle, concentration of the charged particles, length of path between impacts with neutral molecules, and the energy loss at impact. When one attempts, however, to make a complete theoretical investigation of the problem, it becomes apparent that a solution is almost impossible of attainment. With this in mind, we shall first approach the problem from the classical kinetic theory in order to find qualitatively how the various variables affect the motion and then attempt to correct our results by applying certain values obtained experimentally by various investigators.

2. Classical kinetic theory.

We shall assume in this discussion that the electrons, ions and molecules involved are hard spherical particles, that they collide when their centers approach within the distance $\sigma_{12} = \sigma_1 + \sigma_2$, the sum of their radii, and that their motions are described by Maxwell's distribution of velocities.

The relationship of the average kinetic energy in such a system to the mean square speed of the particles is given by

$$E = \frac{1}{2} M c^2 \tag{35}$$

while the average speed and the root mean square speed are related by

$$c = \left(\frac{8\pi}{3} \right)^{\frac{1}{2}} C = .921\, C \tag{36}$$

It should be noted that while the mean energies of admixed gases are equal, the energies of the electrons and ions in a gas cannot be assumed to equal molecular energies.

The average distance a particle moves between collisions with other particles is known as the "mean free path". For uncharged particles, the mean free path of particles of type 1 which collide with particles of type 2 is given by Jeans (5) as

$$\lambda_1 = \frac{1}{\pi N_2 \sigma_{12}^2 \left(1 + \frac{C_2^2}{C_1^2} \right)^{\frac{1}{2}}} = \frac{1}{\pi N_2 \sigma_{12}^2 \left(1 + \frac{E_2 M_1}{E_1 M_2} \right)^{\frac{1}{2}}} \tag{37}$$

When the particles have an electric charge, there is an attraction between them and uncharged particles near by due to the electrical doublet induced in each. This effect is of small magnitude except when the kinetic energies of the particles is small and will therefore be neglected except as empirical data to be introduced later may compensate. The above equation then describes approximately the mean free path of a charged particle. Since the mass of the electron is very much smaller than the mass of the neutral molecules and since as, we will find, the kinetic energy is much higher than that of the neutral molecule, we may simplify and obtain the mean free path of an electron as

$$\lambda_1 = \frac{1}{\pi N_2 \sigma_{12}^{\,2}} \tag{38}$$

The average energy loss by a particle resulting from impact with another particle is described in terms of the average fraction of the energy E_1 lost by a particle of mass M_1 in collision with a particle of mass M_2 whose energy is E_2. Cravath (3) gives for collisions between elastic spheres

$$f = 2.66 \frac{M_1 M_2}{(M_1 + M_2)^2} \left(1 - \frac{E_2}{E_1}\right) \tag{39}$$

but this expression is of little use since the actual collisions differs markedly from that of elastic spheres in the matter of energy loss, particularly above excitation energies, and we must resort to observed data to determine the value.

For the coefficient of diffusion of particles of type 1 through particles of type 2, Compton (2) gives

$$D = \frac{.921\,(c_1^2+c_2^2)^{\frac{1}{2}}}{3\pi\, N\, \sigma_{12}^2} = \frac{4}{(3\pi)^{3/2}\, N\sigma_{12}^2}\left(\frac{E_1}{M_1}+\frac{E_2}{M_2}\right)^{1/2} \qquad (40)$$

where N is the total number of particles of both types per unit volume.

Mobility is defined in terms of the field strength and the average drift velocity in the direction of the field by the relation

$$v = k\,X \qquad (41)$$

even though the mobility is not in general a constant. It's value may be deduced from the diffusion coefficient. Thomson (14) gives the relation as

$$k = \frac{3}{2}\,\frac{De}{E_1} = \frac{3\,De}{M_1 c_1^2} \qquad (42)$$

Substituting the value of D from equation (40) gives us

$$k = \frac{.921\,(c_1^2+c_2^2)^{\frac{1}{2}}}{\pi N\sigma_{12}^2\, M_1 c_1^2} = \frac{2e}{(3\pi)^{\frac{1}{2}}\,\pi N\sigma_{12}^2 E_1}\left(\frac{E_1}{M_1}+\frac{E_2}{M_2}\right)^{1/2} \qquad (43)$$

In so far as collisions may be considered to be between elastic spheres, equations (40) and (43) should permit an evaluation of the mobility and coefficient of diffusion provided we knew the mean velocities and energies. Lacking direct information, we shall attempt a theoretical analysis based on a paper by Compton (2).

A charged particle in a field, moving a distance dx in the direction of the field, gains energy

$$dE = eX\,dx \tag{44}$$

while at the same time losing energy due to collisions with neutral molecules

$$dE = -nfE_1\,dx \tag{45}$$

where n is the average number of collisions made in advancing unit distance. Thus the resulting rate at which the particle receives energy is

$$\frac{dE}{dx} = eX - nfE \tag{46}$$

Terminal speed is reached when the rate of gain of energy equals zero and thus when

$$E = \frac{eX}{nf} \tag{47}$$

The average number of collisions made while advancing unit distance is

$$n = \frac{c_1}{\lambda_1 v} = \frac{.921\,C_1}{\lambda_1 v} \tag{48}$$

Substituting kX for v and the values of λ and k from equations (37) and (43) gives us

$$n = \frac{M_1 C_1^2}{e X} \left(\pi N \sigma_{12}^2\right)^2 \tag{49}$$

which when substituted back in equation (47) gives for the terminal energy

$$E_1 = \frac{e X}{\sqrt{2}\, \pi N \sigma_{12}^2 \sqrt{f}} \tag{50}$$

and for the terminal speed

$$C_1^2 = \frac{2 E_1}{M_1} = \frac{\sqrt{2}\, e X}{M_1 \pi N \sigma_{12}^2 \sqrt{f}} \tag{51}$$

Except as the deviations from the actual values introduced by assuming elastic impact of solid spherical particles must be ascertained, we are now in a position to apply the equations developed above to the motions of actual electrons and ions in a gas.

3. Motions of actual electrons.

Considerations of the mass and kinetic energies of the electron relative to the neutral molecule leads to a simplification of the equations for mobility and coefficients of diffusion such that we arrive at

$$k = \frac{.921e}{\pi N \sigma_{12}^2 m C_1} \tag{52}$$

and

$$D = \frac{.307 C}{\pi N \sigma_{12}^2} \tag{53}$$

where m is the electronic mass.

It will be convenient to define a term λ_0 as the mean free path of an electron at a pressure of 760 millimeter of mercury and zero degrees centigrade such that

$$\lambda_1 = \frac{760}{P} \cdot \frac{T}{273} \lambda_0 \tag{54}$$

or from equation (38)

$$\frac{1}{\pi N_2 \sigma_{12}^2} = \frac{760}{P} \cdot \frac{T}{273} \lambda_0 \tag{55}$$

Combining equation (55) with equations (40), (43) and (51) gives

$$k = \frac{.921 e}{m C_1} \cdot \frac{760}{P} \cdot \frac{T}{273} \tag{56}$$

$$D = .307 C \, \frac{760}{P} \cdot \frac{T}{273} \lambda_0 \tag{57}$$

$$C_1^2 = \frac{\sqrt{2}\, eX}{m\sqrt{f}} \cdot \frac{760}{p} \cdot \frac{T}{273} \lambda_0 \tag{58}$$

If we can determine experimentally the values of λ_0 and f, we will have expressions for the mobility and coefficient of diffusion from which we can obtain numerical values with a fair degree of accuracy.

From equations (41) and (56), we find the drift velocity of an electron in a field will be

$$v = \frac{.921\, e}{m C_1} \cdot \frac{760}{p} \cdot \frac{T}{273} \cdot \lambda_0 X \tag{59}$$

Raether (11) gives a value of $v = 1.25 \times 10^7$ centimeter per second at 295 degrees Kelvin, 760 millimeter pressure, and $X/p = 41$ volts per centimeter per millimeter. Brose and Saayman (1) give the electron free path in air under somewhat similar conditions as $\lambda_0 = 3.6 \times 10^{-5}$ centimeters. Substitution of these values into equation (59) gives for the terminal speed under these conditions $C_1 = 1.57 \times 10^8$ centimeter per second. When substituted into equation (58), this gives a value of $\sqrt{f} = .1225$.

In our study of the arc we shall be concerned with the ratio D/k more than with the mobility and the coefficient of diffusion individually. From equations (42) and (58) we get

$$\frac{D}{k} = \frac{mC_1^2}{3e} = 358\,\frac{X}{p}\cdot\frac{T}{273}\cdot\frac{\lambda_0}{\sqrt{f}} \qquad (60)$$

If we neglect the variation of $\lambda_0/\sqrt{f}$ with X/p, this reduces to

$$\frac{D}{k} = .105\,\frac{X}{p}\cdot\frac{T}{273} \qquad (61)$$

4. Motions of actual ions.

In determining the mobility and coefficient of diffusion of ions in a gas, we must again resort to experimental data if we are to arrive at quantitative results.

By making a substitution of the form

$$\lambda_1 = \frac{p_0}{p}\cdot\frac{T}{T_0}\lambda_0 = \frac{C_1}{\pi N_2 \sigma_{12}^2 (C_1^2 + C_2^2)^{\frac{1}{2}}} \qquad (62)$$

in equations (40) and (43) where λ_0 is now the mean free path of an ion at pressure p_0 and temperature T_0 we find that

$$D = \frac{.307\,(C_1^2 + C_2^2)}{C_1}\cdot\frac{p_0}{p}\cdot\frac{T}{T_0}\lambda_0 \qquad (63)$$

and

$$k = \frac{.921\,e\lambda_0\,(C_1^2 + C_2^2)}{M_1 C_1^3}\cdot\frac{p_0}{p}\cdot\frac{T}{T_0} \qquad (64)$$

In a weak field, the energies of the ions and molecules are roughly equal, and since the masses are equal, the terminal speed C_1 will equal C_2. Equations (63) and (64) reduce for this special case to

$$D = .614\, C_2 \frac{p_0}{p} \cdot \frac{T}{T_0} \lambda_0 \qquad (65)$$

and

$$k = \frac{1.842\, e \lambda_0}{M C_2} \cdot \frac{p_0}{p} \cdot \frac{T}{T_0} \qquad (66)$$

Since, however, C_2 is proportional to the square root of the gas temperature we may write

$$D = \frac{p_0}{p} \left(\frac{T}{T_0} \right)^{3/2} D_0 \qquad (67)$$

and

$$k = \frac{p_0}{p} \left(\frac{T}{T_0} \right)^{1/2} k_0 \qquad (68)$$

where D_0 and k_0 are the coefficient of diffusion and mobility at pressure p_0 and temperature T_0.

Slepian (13) gives a value of the mobility of 1.4 centimeters per second per volt per centimeter at 22 degrees centigrade and a pressure of 760 millimeter of pressure. Introducing this value into equation (68) and converting to electro static units gives

$$k = \frac{760}{p}\left(\frac{T}{293}\right)^{\frac{1}{2}} 420 \quad \frac{\text{cm.}}{\text{sec.}} \Big/ \frac{\text{stat volts}}{\text{cm.}} \tag{69}$$

APPENDIX II*

THE PHENOMENA OF DEIONIZATION

1. The theory of deionization.

Ionized particles in a gas are known to give up their charges and revert to neutral molecules in a number of ways. The most common of these are direct recombination of electrons and positive ions in a gas, recombination between positive and negative ions in a gas or on solid surfaces, and electron attachment to a neutral molecule forming a negative ion with subsequent recombination of the negative ion with a positive ion to form two neutral molecules. These actions are not all equally probable, however, in a gas at a given temperature.

2. Electron attachment to form negative ions.

An electron in a gas at normal temperatures and pressures may attach to a neutral molecule and produce a negative ion. If we let n be the average number of collisions per second of electrons with neutral gas molecules and n' be the average number of impacts that result in one attachment to produce a negative ion, the average time for an electron to form a negative ion will be n'/n. At a pressure of one atmosphere and a temperature of zero degrees centigrade in air, Compton (2) gives values of $n' = 2.0 \times 10^4$

* For a more detailed explanation, see reference (2).

and n $= 3.17 \times 10^{11}$ which gives a time of electron attachment of $.63 \times 10^{-6}$ seconds. At higher temperatures, as in an arc discharge, electron impact is much less probable and free electrons must be assumed to be present.

3. Coefficient of recombination.

If the number of positive and negative ions initially present in a gas are equal, the density of the ionization will decrease after the ionizing conditions are removed according to the relation

$$\frac{dN}{dt} = -\alpha_r N^2 \tag{70}$$

where α_r is a constant known as the coefficient of recombination and is characteristic of the type of ion and the recombination mechanism. If the number of ions present initially is large, this equation becomes upon integration

$$N = \frac{1}{\alpha_r t} \tag{71}$$

where t is measured from the time that the ionizing influences were removed.

The above relationships assumed a random distribution of the ions.

Luhr (9) determined values of the coefficient and gives 1.23×10^{-6} cubic centimeter per second as the value for air at 20 degrees centigrade and a pressure of 760 millimeters of mercury with ions that have been in existance for 5×10^{-2} seconds. A higher value of the coefficient is often quoted for ions immediately after formation. This increase is due to the fact that there has not been time for the ions to diffuse from the positions where they were created and a grouping in pairs of oppositely charged ions exists rather than a random distribution.

It has been developed theoretically and verified experimentally that α_r varies directly as the gas density and inversely as the 3/2 power of the temperature. The coefficient at any temperature and pressure will be

$$\alpha_r = 1.23 \times 10^{-6} \frac{P}{760} \left(\frac{293}{T} \right)^{5/2} \qquad (72)$$

if we accept Luhr's value as correct. Since the temperature of the gas with which we will be concerned will be rather high, free electrons must be assumed to be present. The coefficient of recombination for electrons combining directly with positive ions is much lower than that for recombination between positive and negative ions so that increasing α_r to account for non random distribution does not seem warranted.

APPENDIX III

TABULATED COMPUTATIONS AND SUMMATION OF RESULTS

1. Computations.

In figure (14), values of the maximum field intensity X for the indicated voltage recovery rates and the corresponding values of the ratio $XT/293p$ for various points in the cathode space charge region have been listed. These were obtained by evaluation of equations (29) and (30) and are used to plot the curves of figure (9).

Figure (15) is a listing of the values of α at points in the gap taken from the curve of figure (1) by entering with data from figure (14). The values of α indicated were used to plot the curves of figure (10).

Figure (16) is a tabulation of the solutions of equation (32). These were used to plot the curves of figure (11) from which the minimum value of L was determined.

2. Summation of results.

With the minimum L determined, it was possible by use of equations (28) and (8) to determine the ion density at the streamer tip and the breakdown voltage. These values are tabulated in figure (17) and form the basis for the curve of figure (12)

Tabulation of XT/293p for successive values of B

B =	$10x10^6$	$25x10^6$	$25x10^6$	$40x10^6$	$60x10^6$	$80x10^6$
T =	1273	1273	2273	1273	1273	1273
X_o =	9707	15348	14515	19414	23777	27435
x/L	**	**	**	**	**	**
0.0	55.49	87.74	148.05	110.99	135.93	156.85
0.1	52.65	83.25	140.46	105.31	128.97	148.82
0.2	49.65	78.51	132.42	99.31	121.63	140.35
0.3	46.46	73.46	123.88	92.92	113.80	131.31
0.4	43.03	68.03	114.68	86.06	105.40	121.62
0.5	39.30	62.14	104.70	78.60	96.27	111.08
0.6	35.15	55.62	93.64	70.36	86.17	99.43
0.7		48.22	81.10	61.00	74.71	86.20
0.8		39.47	66.24	49.93	61.15	70.57
0.9		28.12	46.86	35.37	43.57	50.27
XT/293p =	34.8	34.8	37.2	34.8	34.8	34.8
x/L =	.575	.875	.960	.910	.935	.960

Solutions of equations (29) and (30)

Figure 14

Tabulation of α for successive values of B

B =	10×10^6	25×10^6	25×10^6	40×10^6	60×10^6	80×10^6
T =	1273	1273	2273	1273	1273	1273
X_o =	9707	15348	14515	19414	23777	27435
x/L	**	**	**	**	**	**
0.0	15.04	76.10	159.70	146.93	232.64	297.36
0.1	11.54	66.50	141.10	124.19	201.16	279.89
0.2	9.45	56.10	125.40	106.70	181.92	253.63
0.3	7.17	45.50	106.80	90.96	153.93	218.65
0.4	4.55	35.50	88.20	73.47	126.82	183.67
0.5	2.54	24.50	69.60	56.85	99.70	146.93
0.6		14.90	51.40	40.23	73.64	106.70
0.7		8.40	34.70	23.61	47.93	74.34
0.8		2.62	17.90	9.08	23.26	40.23
0.9			4.10	1.24	5.00	9.10
α =	1.00	1.00	1.00	1.00	1.00	1.00
x/L =	.575	.875	.960	.910	.935	.960

Values of α obtained from curve of figure (1)

Figure 15

Solution of equation (32)

x/L	15.3478	$-\log(\alpha/100)$	$1/2\log(x/L)$	$\log(X_s/17492)$	Σright side	$\int_0^{x/L}\alpha\, d(x/L)$	L
0.1	"	2.1594	-1.1513	-.6415	15.7144	1.318	14.3325
0.2	"	2.3592	- .8047	-.7002	16.2021	2.454	6.9988
0.3	"	2.6353	- .6020	-.7666	16.6145	3.318	5.2575
0.4	"	3.0900	- .4578	-.8433	17.1358	3.954	4.5246
0.5	"	3.6730	- .3465	-.9334	17.7404	4.545	4.0573
0.575	"	4.6025	- .2767	-1.0556	18.6207	4.773	4.0477

$B = 10\text{x}10^6$; $T = 1273\ °K$

Figure (16a)

Solution of equation (32)

x/L	15.3478	$-\log(\alpha/100)$	$\log(X_s/17492)$	$1/2\log(x/L)$	Σright side	$\int_0^{x/L}\alpha\, d(x/L)$	L
0.1	"	.4080	- .1833	-1.1513	14.421	7.198	2.054
0.2	"	.5780	- .2419	- .8047	14.879	13.279	1.125
0.3	"	.7850	- .3084	- .6020	15.225	18.360	.8238
0.4	"	1.0498	- .3852	- .4587	15.554	22.378	.6868
0.5	"	1.4065	- .4758	- .3465	15.932	25.453	.6165
0.6	"	1.9038	- .5866	- .2554	16.410	27.371	.5899
0.7	"	2.4769	- .7294	- .1783	16.917	28.475	.5847
0.8	"	3.6420	- .9296	- .1115	17.949	28.940	.6117
0.875	"	4.6052	-1.0556	- .0667	18.835	29.102	.6395

$B = 25\times10^{6}$; $T = 1273\ ^{\circ}K$

Figure (16b)

Solution of equation (32)

x/L	15.0539	-log(α/100)	1/2log(x/L)	log(X_s/9798)	Σ right side	$\int_0^{x/L} \alpha\, d(x/L)$	L
0.1	"	- .3443	-1.1513	.3396	13.9033	15.86	.8723
0.2	"	- .2263	- .8047	.2808	14.3091	29.14	.4784
0.3	"	- .0658	- .6020	.2141	14.6056	40.58	.3469
0.4	"	.1256	- .4587	.1370	14.8632	50.23	.2834
0.5	"	.3624	- .3465	.0459	15.1211	58.28	.2476
0.6	"	.6655	- .2554	- .0657	15.4037	64.35	.2279
0.7	"	1.0584	- .1783	- .2095	15.7299	68.65	.2180
0.8	"	1.7204	- .1115	- .4119	16.2565	71.28	.2174
0.9	"	3.1942	- .0527	- .7580	17.4428	72.23	.2314
0.960	"	4.6052	- .0204	- .9889	18.6552	72.41	.2480

$B = 25 \times 10^6$; $T = 2273\,^\circ K$

Figure (16c)

Solution of equation (32)

x/L	15.3478	$-\log(\alpha/100)$	$1/2\log(x/L)$	$\log(X_s/17492)$	Σright side	$\int_0^{x/L}\alpha d(x/L)$	L
0.1	"	- .2166	-1.1513	.0517	14.0316	13.364	1.052
0.2	"	- .0649	- .8047	- .0069	14.4713	25.046	.5665
0.3	"	.0598	- .6020	- .0734	14.7672	34.819	.4114
0.4	"	.3083	- .4587	- .1501	15.0473	43.137	.3362
0.5	"	.5648	- .3465	- .2408	15.3253	49.910	.2948
0.6	"	.9106	- .2554	- .3515	15.6515	54.592	.2749
0.7	"	1.4435	- .1783	- .4943	16.1187	56.292	.2749
0.8	"	2.3991	- .1115	- .6945	16.9409	57.701	.2827
0.9	"	4.3901	- .0527	-1.0337	18.6515	58.110	.3109
0.910	"	4.6052	- .0471	-1.0556	18.8503	58.200	.3139

$B = 40\times10^{6}$; $T = 1273\ ^\circ K$

Figure (16d)

Solution of equation (32)

x/L	15.3478	$-\log(\alpha/100)$	$1/2\log(x/L)$	$\log(X_s/17492)$	Σ right side	$\int_0^{x/L}\alpha\, d(x/L)$	L
0.1	"	- .6989	-1.1513	.2544	13.752	21.484	.6294
0.2	"	- .5984	- .8047	.1958	14.1403	40.492	.3359
0.3	"	- .4313	- .6020	.1293	14.4438	57.453	.2389
0.4	"	- .2376	- .4587	.0526	14.7041	71.733	.1935
0.5	"	.0030	- .3465	- .0380	14.9663	83.323	.1689
0.6	"	.3060	- .2554	- .1488	15.2496	92.021	.1556
0.7	"	.7354	- .1783	- .2916	15.6131	97.113	.1505
0.8	"	1.4584	- .1115	- .4918	16.2029	100.512	.1518
0.9	"	2.9957	- .0527	- .8308	17.4600	101.943	.1624
0.935	"	4.6052	- .0340	-1.0560	18.8630	102.049	.1763

$B = 60\times10^6$; $T = 1273\ ^\circ K$

Figure (16e)

Solution of equation (32)

x/L	15.3478	-log(∝/100)	1/2log(x/L)	$\log(X_s/17492)$	Σ right side	$\int_0^{x/L} \alpha\, d(x/L)$	L
0.1	"	-1.0292	-1.1513	.3976	13.5649	29.091	.4527
0.2	"	- .9307	- .8047	.3392	13.9514	55.773	.2373
0.3	"	- .7823	- .6020	.2724	14.2359	79.727	.1673
0.4	"	- .6080	- .4587	.1957	14.4768	99.681	.1352
0.5	"	- .3848	- .3465	.1051	14.7216	116.045	.1176
0.6	"	- .0649	- .2554	- .0057	15.0218	128.873	.1079
0.7	"	.2965	- .1783	- .1485	15.3175	138.136	.1026
0.8	"	.9106	- .1115	- .3486	15.7983	143.727	.1020
0.9	"	2.3969	- .0527	- .6878	17.0042	148.091	.1073
0.960	"	4.6052	- .0204	-1.0556	18.8770	148.455	.1200

$B = 80\times10^{6}$; $T = 1273\ ^\circ K$

Figure (16f)

TABULATION OF FINAL RESULTS

B =	10×10^{6}	25×10^{6}	25×10^{6}	40×10^{6}	60×10^{6}	80×10^{6}
T =	1273	1273	2273	1273	1273	1273
L =	4.03	0.5835	0.2165	0.2735	0.1500	0.1010
t =	2615.52×10^{-6}	239.48×10^{-6}	100.53×10^{-6}	88.74×10^{-6}	39.73×10^{-6}	23.17×10^{-6}
x/L =	0.535	0.665	0.760	0.650	0.750	0.775
V =	26155.2	5987.0	2513.8	3549.6	2383.8	1853.6
N =	5.98×10^{10}	14.0×10^{10}	13.6×10^{10}	23.3×10^{10}	33.5×10^{10}	43.0×10^{10}

Figure (17)